TRADING BEHAVIOR IN THE FOREIGN EXCHANGE MARKET

BY

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I dedicate this dissertation to my parents, Franz and Christel Kauffmann.

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This study describes the microstructure of the foreign exchange market. It emphasizes the decision-making process of typical foreign exchange dealers. The dealers, who are exposed to randomness in both the customer and the interdealer markets, follow time-invariant, optimal pricing and trading strategies. These strategies assign to each combination of dealer inventories those rates and orders which maximize the expected present value of the dealers' current and future profits. They consider the competitors' optimal behavior and the fact that each dealer's decision-making process is sequential. Two infinite-horizon dynamic programming models are developed which describe the operation of a two-dealer market.

The first model ignores interdealer trading completely. It assumes that dealers merely trade with their customers and hence are only indirectly related over price competition. In an infinite-horizon framework each dealer's optimal pricing strategy is derived and a noncooperative Nash equilibrium is found for the market as a whole, using a linear-quadratic framework. Due to the complexity of the

solution, Nash equilibria for finite versions are also developed to assist in interpretation of the outcomes. For the models with one and two periods in the horizon a negative relation between a dealer's optimal exchange rate and both dealers' inventories is derived.

Interdealer trading is introduced in the second model. Each dealer now determines the amount of foreign exchange to trade with his competitor at the same time as choosing his own exchange rate. The noncooperative Nash equilibrium for this model, which consists of both dealers' optimal pricing and ordering strategies, is also derived. A one-period horizon version of this infinite-horizon model is presented to interpret outcomes. As before, each dealer's optimal pricing strategy is negatively related to both dealers' inventories. Furthermore, an inverse relationship between a dealer's optimal ordering (purchase) and his inventory is derived.

CHAPTER I
INTRODUCTION

The usefulness of macroeconomic models for explaining the determination and movement of the foreign exchange rate in an open economy cannot be doubted. They express the exchange rate as a function of various aggregate variables, for instance, the relative income level, the relative money supply, the nominal interest differential, the expected inflation differential and the current account level. Studies by Frankel (1979), Calvo and Rodriguez (1977), Kouri (1976), and Dornbusch and Fischer (1980) are examples from the extensive body of literature. Its growth has been accelerated since Dornbusch (1976) introduced rational expectations into international monetary economics.

Meese and Rogoff (1983a, 1983b) compared the out-of-sample forecasting accuracy of several of these structural models and time-series exchange rate models. They found that at one- to twelve-month horizons the random walk model performed as well as any other estimated model. They also showed that the relative superiority of the random walk model over the structural models decreases as the forecast horizon moves towards twelve months. In his comment on their paper (1983b), Salemi (1983, p. 111) found "it an interesting possibility that, in the short run, the spot rate behaves like a price of a speculative asset but that over longer horizons its equilibrium value is systematically related to other economic variables, as the asset models predict." Following this idea it seems conceivable that a microeconomic model

describing how exchange rates are determined in the short run to equilibrate flow supply of and demand for foreign exchange would provide some insight in their short-run behavior.

Descriptions of the microstructure of the foreign exchange market, picturing the agents in this market, their actions and the resulting formation of exchange rates, have been only of verbal nature (e.g., Coninx, 1980; Einzig, 1966; Kubarych, 1983; Riehl and Rodriguez, 1983; Tygier, 1983; Wasserman, Prindl and Townsend, 1972). The lack of mathematical models in this area is usually not attributed to any doubt about their potential ability to simplify the analysis. It is rather ascribed to the complexity of the foreign exchange market and the resulting difficulty of capturing its most important characteristics in a tractable manner.

Facing these problems, I also initially resort to words to describe the complex functioning of the foreign exchange market before actually modeling it. Especially the reader unfamiliar with this particular market should find this approach conducive to a better understanding of the models presented in the following chapters.

A foreign exchange market is usually not a market in the physical sense, but rather a mechanism of exchange of low interest financial papers denominated in different currencies. This market type is called "over-the-counter market" (cf. Kubarych, 1983). The need for it arises from international trade as well as from international financial investment. The few other forms in which currencies are traded against each other, e.g., the bank notes tourists buy from banks, are of minor importance. The connection between individual agents is made mostly by telephone, but also by telex and other telecommunication systems which

are used depending on the respective costs, the urgency of the planned transaction, and the preference of its initiator. The development of speedy communication has reduced the time within which a transaction is made, including the inquiry, quotation and acceptance, to a few minutes if not seconds.

There are several groups of agents in this market which are unfortunately not as homogeneous as would be desirable. These are the foreign exchange dealers, their customers, the foreign exchange brokers and the government.

The foreign exchange dealers buy from and sell foreign currency to customers and other banks. The bulk of the foreign exchange transaction in each currency is conducted by a few large banks located in (one of) the financial centers of a country. The 1983 US Foreign Exchange Market Turnover Survey conducted by the Federal Reserve Bank of New York concluded that the ten most active banks accounted for roughly 40 percent of all transactions in the US in April 1983. Commercial banks who deal in foreign exchange only occasionally to satisfy their customers are considered "financial customers." The typical foreign exchange dealers specialize in two or more currencies. For these currencies they are "market-makers"; i.e., they are prepared to trade them at any time with other market-making banks. For simplicity it seems reasonable to concentrate on one foreign currency only, say, the Deutsche Mark which, according to the 1983 turnover survey mentioned above, was with 32.5 percent of the banks' foreign exchange transactions the most actively traded currency in the United States. In the 1986 turnover survey its share even reached 34.2 percent. Any risk trade-off which can be achieved by "making the market" in two currencies will be

ignored. During my visit to two banks in the New York Foreign Exchange Market I found that this assumption of specialization in one foreign currency is quite realistic for some banks.

Although all the banks under consideration presumably deal in the same currency, they are a rather heterogeneous group. Some concentrate on forward exchange (where the delivery of the currency is at a future date); some concentrate on spot exchange or both. Forward and spot rates are linked by the interest differential, but their difference, the "premium" or "discount," also reflects expectations. The present study will concentrate solely on the spot market which accounted for roughly 63 percent of the total foreign exchange transactions in the US in April 1986 as opposed to 30 percent for the swap transactions (exchanging a spot for a forward value or two forward values against each other) and roughly 5 percent for outright forward transactions, according to the 1986 turnover study.

In addition, although all the banks maximize their overall discounted current and future profits (which includes profits in the money market as well), they emphasize differently the intermediate (foreign exchange market) goals of providing good customer service and maximizing the direct profits from the foreign exchange market (cf. Hudson, 1979). This difference results chiefly from divergent customer structure. If, for example, the bank earns large profits in the money market from some multinational corporation, it will be willing to incur losses from a transaction in the foreign exchange market with that customer if it can ensure thereby the corporation's loyalty to the bank.

Nevertheless, to keep the model as simple as possible, the assumption will be made that the dealers maximize the foreign exchange

department's profits independently from those of the rest of the bank. The difference in customer structure and in overall bank management will be taken into account by assuming different excess demand functions for foreign exchange and different overnight holding costs of the two dealers. The latter discrepancy is caused by a difference in internal interest rates which are charged by the bank management to the foreign exchange department as the opportunity cost of funds held in the form of foreign exchange (cf. Wasserman, Prindl and Townsend, 1972). But even if the dealer functions were assumed to be identical, the two dealers need not hold the same inventory or quote the same rates at any instant, because they would still be exposed to different random excess demand shocks.

The customers of the foreign exchange dealers, who can be considered the end-users of the foreign exchange market, individually have a relatively small effect on what happens in the market (in the case of multinational corporations there are exceptions). Important customers are the exporting and importing firms whose main interest lies in the production and sale of their goods (thus not in capital gains from exchange rate movements) and investors and borrowers who try to find the highest yield on funds lent or the lowest cost of borrowing funds either at home or abroad. Speculators, on the other hand, try to make capital gains from the movement of the exchange rate based on their own expectations. Wasserman, Prindl and Townsend (1972) argue that ordinarily the volume of pure speculation falls short of "that of international business transactions in hedging and speculative adjustment of foreign exchange positions." Despite the heterogeneity

described above, from here on the customers will be referred to as one homogeneous group.

The third group of agents in the foreign exchange market are the brokers. From the 1986 turnover survey quoted earlier it can be concluded that there are about nine foreign exchange brokers in the United States. Their function is to bring banks who want to buy or sell a currency together with a counterpart. They charge a brokerage fee which is a constant amount per unit transacted. Since 1978, the buying and the selling bank each pays half of this fee. Before that the seller of the foreign exchange had to pay the whole commission (compare Kubarych, 1983).

It is important to note that the brokers do not trade for their own account; i.e., they do not hold inventories in the currencies they are trading. This represents a major difference from the behavior of the specialist in Bradfield and Zabel's paper "Price Adjustment in a Competitive Market and the Exchange Specialist" (1979), an interesting study of the behavior of a price-maker (the specialist) in a competitive (the stock) market that inspired the present paper. Bradfield and Zabel's specialist maximizes his profit by setting a price and taking a position in the market in response to expected and actual market and limit orders which are submitted by the traders. In the foreign exchange market, on the other hand, the brokers merely bring interested buyers and sellers together and inform the foreign exchange dealers about the distribution of the exchange rate quotations in the market. Based on this information and on their present inventory positions, the foreign exchange dealers then choose their own quotations which maximize

the expected dealer profits. Since 1978 the brokers are no longer confined to arranging transactions between US banks but they can also arrange deals between US and foreign banks or even between two foreign banks. On the other hand, US dealers can now deal directly among themselves whereas before they only dealt directly with foreign banks (cf. Revey, 1981).

Before describing the foreign exchange dealer's activity, let me mention the last important agent, the government. While the government intervened constantly under a fixed exchange rate system as the Bretton Woods System, under the now prevailing system of "managed floating" its role is merely to keep an "orderly market." This means that it is available as a trading partner in case of a one-sided market (for instance, where there are only buyers and no sellers) or that it might try to dampen momentary fluctuations. An announced intervention in the foreign exchange market changing the customers' and dealers' expectations as opposed to an unannounced intervention will have different effects on the behavior of the agents in the foreign exchange market. But it will be assumed here that the government does not intervene at all, although the distinction should be kept in mind for further research.

It is now time to take a closer look at a typical market-maker and, specifically, at the dealing department's activity. As mentioned above, the dealing department will be considered as an independent part of the bank in the sense that it tries to maximize its profits independently. Profits arise from two sources. The first is capital gains from taking a position to exploit an expected movement in the exchange rate. An example would be to buy a certain amount of foreign exchange at a time

when its dollar equivalent is relatively low in order to sell it later for a higher amount of US-dollars.

The second source of profit is the spread. The spread is the difference between the ith dealer's selling and buying or the ask and bid prices, p_i^a and p_i^b. In terms of the above symbols, the ith dealer's exchange rate p_i,[1] to which I exclusively have been referring so far, is then $(p_i^a + p_i^b)/2$. Several papers in the finance literature discuss the determination of the spread as a price of the provision of immediacy by the dealers (see for example Cohen, Maier, Schwartz and Whitcomb, 1979, Demsetz, 1968, and Zabel, 1981). Since empirically the spread tends to be stable, it will be assumed constant throughout the present study. Furthermore, given this assumption, I will ignore the second source of profits completely, assuming that the ith dealer chooses only a single price p_i, at which he is willing to buy and sell foreign currency. Bradfield and Zabel show for their stock market model that in both approaches, with and without spread, the equilibrium inventories and price variances are identical. The equilibrium prices in their model with bid and ask prices turned out to be simple translates of that with only one price as the choice variable. Since my main objective is to explain the movement of a dealer's average exchange rate and not his total profits, this concentration on capital gains only seems to be justified. A third potential source of income, the dealer's involvement in the money market with the objective to invest the bank's funds most profitably and to borrow in the cheapest manner will also be ignored. (For a discussion on the subject see Hudson, 1979.)

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[1] The exchange rate p_i used in this study is defined as the amount of foreign exchange per dollar. That means "direct quoting" is used as opposed to quoting in "European terms."

Although there is a hierarchy of dealers in the dealing department, starting from the trainee up to the chief or principal dealer (cf. Coninx, 1980; Tygier, 1983, I will henceforth refer to "the dealer" as the one person responsible for the dealing department, which most closely reflects the chief dealer's function.

The dealing department, or simply the bank, holds an inventory of foreign currency in the form of working balances with its foreign branches or correspondents in the country in whose currency it specializes. These balances are held in so-called "nostro" (the Latin word for "our") or "due from" accounts. The dealer is almost continuously buying and selling foreign exchange, either passively or actively, depending on whether he himself or his counterparts initiate the trade. The counterparts in these transactions are customers and other dealers (in the case of passive trading), or only dealers (in the case of active trading).

If the dealer's trading horizon is subdivided into a finite number of periods of equal length, e.g., one day, at the beginning of each period the dealer would face the same type of situation. He knows the inventory the bank is holding in the foreign currency. He has some notion of the total flow demand for and supply of the foreign currency by all customers at different exchange rates during this period. In addition, knowing his competitors' quotations, or even their reactions to his own quotations or other events, he has some idea of the individual demand and supply function he is facing from the customers during that period. The rather vague expressions "some notion, some idea" will later be replaced by the assumption that he knows the nonstochastic part of the demand and supply functions and the

distribution of the random part but not its future realizations. In addition, he expects some buy and sell orders from other banks. Limit orders are ignored at this point. The dealer has to consider that certain costs will be incurred at the end of the day. The internal accounting system of a bank charges an internal interest at a rate slightly above the current rate for bills or deposits for the dollar equivalent of the foreign currency held over night. From these costs he can subtract the very low interest which the foreign bank pays on the inventory in the "nostro" account. Should the dealer borrow funds over night from some other bank, the interest rate charged will be above the internal rate charged for the funds tied up.

Given all this information and given his expectations about the future development of the market, the dealer has to decide two things simultaneously. First he has to make a decision whether he wants to change his current inventory by buying from or selling to another dealer at that dealer's rate. Second he must choose an exchange rate. When the dealer makes this choice he will not just seek to maximize the expected profits of this one-day period he is in, but he must also keep in mind that a day later he will have an analogous decision problem and a day later again, and so on. He will take into account that the current decision will affect his inventory at the beginning of the next period. This implies that the dealer will try to maximize his profits over an infinite horizon by choosing the amount by which he wants to change his inventory and by choosing the current price, keeping in mind that he will be able to make an analogous decision at the beginning of every future period. Under the assumption that the expected excess

demand in all markets does not change, which is, I must admit, in the foreign exchange market a quite unrealistic approach, the foreign exchange dealer can work out an optimal strategy which tells him at the beginning of each period which change in inventory and which exchange rate to choose to maximize his expected infinite-horizon profits for a given starting inventory. This strategy is complicated by the fact that the quotations of the competitors given at the beginning of the next period may have changed due to a random shock in their excess demand of the current period. Considering this additional problem I must agree with Rothschild's (1973, p. 1291) statement "it is a formidable task to design rules appropriate for dynamic, uncertain, and at least momentarily imperfectly competitive situations." Nevertheless, the remainder of this paper will demonstrate that the task to develop an optimal strategy for a typical dealer in the foreign exchange market, as described above, is maybe tedious, but not impossible to accomplish.

The following chapters present two basic models. The models have in common that they are both dynamic programming models (see Bellman, 1957; Bradfield and Zabel, 1979), which take into account the interaction between two dealers. The former represents the dynamic and uncertain aspect of this market, while the latter symbolizes that the market consists of more than one price-maker. The decision to use a two-dealer model (rather than modeling the trading behavior of more than two foreign exchange dealers) was derived from the need for a tractable model that would still be able to reflect the game-theoretic aspect. In addition, it is often the case that a two-agent model can be extended to one with a finite number $d > 2$ of agents without changing the results

dramatically. Both models consist of two value functions which represent the behavior of two typical foreign exchange dealers. Both dealers maximize an expected profit function over an infinite horizon by choosing optimal rates sequentially.

The first model, called the "Model Without Interdealer Market," is discussed in the second chapter. As the name already indicates, this model ignores interdealer trading completely. The two dealers buy from and sell to customers, but do not trade with each other. Nevertheless, the two dealers' activities are connected since one dealer's rate enters as an argument in the other's excess demand function. The noncooperative Nash equilibrium consists of the two dealers' optimal pricing strategies, each of them functions of both dealers' inventories. To assist in interpreting the solution to the infinite-horizon model, the Nash equilibrium for finite versions of the model (with one and two periods in the horizon) are analogously derived.

Interdealer trading is introduced in the third chapter where the "Model With Interdealer Market" is discussed. In addition to the exchange rate, each dealer now has a second decision variable called "ordering." This represents the fact that each dealer has to determine the amount he wants to buy from or sell to the other at the same time as he chooses his own exchange rate. That decision-making process is also repeated at the beginning of every period. As before, a Nash equilibrium is derived for this model. It consists again of the two optimal pricing strategies as functions of both inventories. In addition, two optimal ordering strategies are now part of the noncooperative solution. Their arguments are also both dealers'

current inventories. Again, a finite version (one period in the horizon) of this infinite-horizon model is presented and its Nash equilibrium derived.

The fourth chapter contains the conclusions of the present study. It summarizes the results and includes suggestions for further research.

CHAPTER II
THE MODEL WITHOUT INTERDEALER MARKET

Introduction of a Dealer's Decision-Making Process

As described in the introduction, trading in the foreign exchange market is an ongoing process. The current situation under which a dealer has to make a decision is the outcome of his earlier actions as well as that of past events beyond his control. By the same token, the dealer's current decision will influence but not completely determine the circumstances under which future decisions have to be made. Recognizing this intertemporal connection between a foreign exchange dealer's subsequent decision-making processes, a representation of his trading behavior in terms of a dynamic programming model seems appropriate.

In addition to the link over time between a dealer's trading decisions, it is important to recognize the interrelation between the decision-making processes of all dealers trading in the foreign exchange market. Each dealer is aware of that link and takes it into consideration when making his decisions. To incorporate this game-theoretic aspect of the foreign exchange market into my study while keeping it as simple as possible, I will focus my attention on two foreign exchange dealers only.

There exist two points of connection between these two dealers. The first link is provided by the customer market, where both dealers buy and sell foreign exchange. Their customers tend to compare both dealer rates and trade with the dealer who offers the more advantageous

rate in their eyes. Of course, gathering the information costs both time and money. Time is especially important in this market, because a dealer does not guarantee a rate he has quoted to a particular customer while the customer is contacting another dealer. Thus the customer might lose a good trade while trying to get a second quote. This problem of how much information to gather before making a final trade is generally called the economics of search. A thorough discussion on the subject has been provided by Stigler (1961), McCall (1970), and MacMinn (1980) among others.

Compared to the link between both foreign exchange dealers through the customer market, the second connection is more direct. It is given by the trading of foreign exchange between the two dealers in the interdealer market. To be able to distinguish the effect the interdealer market has on trading behavior, I will consider a model without interdealer trading first. In the following chapter trading between the two foreign exchange dealers will then be introduced and the two models will be compared.

But let me now concentrate on the model without interdealer trading. The assumptions in this model are the following. The agents in the market are two foreign exchange dealers and their customers. The brokerage system is not explicitly taken into account, although I assume that each dealer learns from the brokers what rate his competitor is quoting. In addition, the existence and intervention of the government are ignored, or at least not explicitly assumed.

Considering the behavior of dealer i (where $i = 1,2$), I assume the following. The number of trading days in his horizon is infinite. Each trading day constitutes one period in the dealer's infinite horizon. At

the beginning of each period the ith dealer decides which exchange rate to quote. He bases his decision on his own current inventory of foreign exchange and the knowledge of his competitor's optimal pricing strategy. The noncooperative Nash equilibrium of the present model will be derived solving for both dealers' optimal pricing strategies simultaneously.

It is important to keep in mind that the current model does not allow the dealers to trade with each other. This implies that each dealer can only influence his inventory by choosing an exchange rate, not by actively buying or selling foreign exchange. During the period buy and sell orders will arrive from customers, which can be summarized in an excess demand function of the form

$$(2.1) \qquad ED_i = d_i + a_i p_j + A_i p_i + \zeta_i,$$

where i, j = 1,2 (i≠j), ED_i is the excess demand, and d_i, a_i and A_i are constants. It is assumed that A_i is negative and that a_i and d_i are positive. The intuition behind the first assumption is that customers will tend to demand more and supply less foreign exchange to a dealer the lower the rate he is quoting, and conversely for higher rates. The second assumption stands for the effect the competitive dealer's pricing behavior has on a dealer's excess demand. A dealer can expect to gain more potential sellers and lose more potential buyers the lower the foreign exchange rate the other dealer is quoting. A positive d_i represents the fact that the expected excess demand at zero prices is positive. Furthermore, I assume that $|A_i|$ is greater than a_i and a_j. The rationale behind this assumption is first that a dealer's excess demand function should be affected to a greater extent by his own price

than by that of his competitor due to customer loyalty. Second it means that a change in price p_i should have a greater influence on the ith dealer's excess demand than on the jth dealer's excess demand. The last term in the above expression is ζ_i, the additive, serially independent random part of the excess demand function. It is characterized by the joint distribution function $\Phi_{ij}(\zeta_i,\zeta_j)$, the marginal distribution function $\Phi_i(\zeta_i)$ with expected value $E(\zeta_i)=0$, and finite variance $\sigma_{\zeta i}^2$ and covariance $\sigma^2_{\zeta i \zeta j}$. The two dealer rates are, of course, given by p_i and p_j. It should be noted that negative prices have not been excluded. This simplifying provision guarantees that a solution to this problem will be interior. However, future research should also include considerations of nonnegative prices and corner solutions.

The costs that are explicitly taken into account are those that arise from the overnight holding of foreign exchange and fixed costs in operating the exchange department. The latter are less important since they do not affect optimal decisions. The exclusion of the brokerage system also obviates the consideration of costs incurred per unit of foreign exchange traded, as does the omission of the spread, or, jointly, the omission of a "net" spread (spread minus brokerage costs). The costs actually taken into account are the "holding costs," a term borrowed from inventory theory (cf. Arrow, Karlin and Scarf, 1958). They depend on the inventory of foreign exchange held at the end of the current period, which is identical to the inventory held at the beginning of the next period. Following Bradfield and Zabel (1979), I assume that the holding costs are

$$(2.2) \qquad H_i(w_i) = h_{ai} + h_{bi}w_i + h_{ci}\frac{w_i^2}{2},$$

where w_i is the ith dealer's inventory at the end of the period. It is assumed that h_{ai} and h_{ci} are positive, while h_{bi} is negative. The overhead costs are absorbed into the constant term h_{ai}. Although the relationship is not shown explicitly here, the constants h_{bi} and h_{ci} are based on three rates. The first is the internal interest rate charged by the bank to the dealing department for the missed opportunity of investing the dollar equivalent of the inventory holding in the money market. From this rate the low overnight interest paid by the foreign bank where the currency is held in the "nostro" account can be deducted. These two rates are relevant if the overnight inventory of foreign exchange is positive. On the other hand, if it is negative, the foreign exchange dealer will have to borrow foreign currency at a rate which will most likely be above the internal interest rate, but certainly above the difference between the two rates mentioned above. This explains the negativity of h_{bi}.[1] The quadratic form of the holding costs represents the fact that the bank management tries to discourage large foreign exchange risk exposure over night by increasing the internal rate with increased exposure, as well as the increased marginal costs of borrowing and the increased marginal revenue of lending foreign exchange abroad which can be observed in the real world.

Given the above information and adding the assumption that the ith dealer maximizes his expected infinite-horizon profits by choosing his rate p_i sequentially, the dealer's trading behavior can now be formalized. For this purpose, use is made of a stochastic dynamic programming functional equation of the form

[1]The unlikely case where w_i is close enough to zero that $h_{bi}|w_i| + h_{ci}w_i^2/2 < 0$ will be ignored here.

$$V^i(x_i,x_j) = \max_{p_i} \{p_i(d_i+a_ip_j^*+A_ip_i) - \int H_i(x_i-d_i-a_ip_j^*$$

$$(2.3) \qquad -A_ip_i-\zeta_i)d\Phi_i(\zeta_i) + \alpha\int V^i(x_i-d_i-a_ip_j^*-A_ip_i$$

$$-\zeta_i,x_j-d_j-a_jp_i-A_jp_j^*-\zeta_j)d\Phi_{ij}(\zeta_i,\zeta_j)\},$$

where $p_j^*=p_j^*(x_i,x_j)$ is the jth dealer's optimal pricing strategy. The first term on the right-hand side is the expected revenue in the current period. It can also be written as $\int p_i(d_i+a_ip_j^*+A_ip_i+\zeta_i)d\Phi_i(\zeta_i)$. Noting that the ith dealer knows the excess demand he will face in the current period only up to a random term, the best he can do is to use his knowledge of the distribution of this random variable to determine the expected value of his revenue as a function of p_i and p_j^*. Obviously the expected revenue in the current period can be negative or positive as the expected demand can also be either one, depending whether more currency is bought or sold by the foreign exchange dealer.

The second term gives the expected holding costs, where $w_i = x_i-d_i-a_ip_j^*-A_ip_i-\zeta_i$.[2] The argument of the holding cost function is the inventory at the end of the current period, which is, of course, the difference between the starting inventory and the excess demand of the current period. Since the dealer does not know the realization of the random part ζ_i of the excess demand when he makes a price decision in the current period, the best he can do in this situation is to compute

[2] In the stock market model by Bradfield and Zabel (1979) such an inventory term considered for the last of several periods in the day made the specialist become more sensitive to deviations of the actual from his optimal inventory towards the end of the day and less willing to take speculative positions. Although I do believe that the same result would be obtained in the foreign exchange market, if more than one period per day were considered, descriptions of the foreign exchange market (e.g., Einzig, 1961, among many others) have convinced me that it is the potentially strong fluctuations over night rather than the holding costs that the dealers fear.

the expected holding costs as a function of p_i and p_j^*. Since holding costs are an expense incurred by the foreign exchange dealer, the second term is subtracted from the expected revenue of the current period.

The third term, involving the value function, represents the fact that the dealer takes into consideration an infinite horizon when making his current pricing decision and that he will be able to choose other quotes at the beginning of each new period. This type of decision-making is generally called closed loop control (cf. Intrilgator, 1971) or optimal feedback control (cf. Dreyfus and Law, 1977). The arguments of the value function are next period's beginning inventories of foreign exchange for both dealers. Since the net addition to the inventories is partially random, the dealer has to use his knowledge of the joint distribution of ζ_i and ζ_j to determine the expected value of the value function. However, it should be noted that the determination of the expected value of this term is much more complicated than that of the revenue and holding cost terms, because it involves the present value of expressions that reach into infinity. In other words, I need to determine properties of the value function $V^i(x_i,x_j)$ to compute the expectation. The discount rate α (where $0<\alpha<1$) in front of the expected value of the value function represents the fact that the foreign exchange dealer values earlier returns more than later returns.

As indicated above, the ith dealer's only choice variable is p_i. Given p_j^* as a function of x_i and x_j, he will therefore determine the expected present value of all future net profits for various choices of p_i and quote the rate that maximizes this present value. Since such an optimal rate can be found for all possible values of x_i and x_j, he can derive an optimal pricing strategy as a function of x_i and x_j.

The Optimal Pricing Strategy

The Intuitive Result

Before deriving and analyzing this optimal pricing strategy mathematically, let me use my knowledge of the foreign exchange market to conjecture its form. First I consider how a foreign exchange dealer's exchange rate is related to his inventory position of foreign exchange. Does he tend to quote a higher rate the higher or the lower his inventory? Intuition tells me that a high inventory will cause the dealer to quote a low exchange rate, while a low, even negative, inventory position will make him quote a high rate. This, of course, holds true only if the dealer does not have a clear idea as to future movements in the foreign exchange market. Under some circumstances he might want to hold a large positive or negative inventory to exploit anticipated changes in the market. But, to quote Einzig (1966, p. 60), "if they [the dealers] hold no definite views about the prospects of the exchange rate they may adhere to the textbook rule of covering the spot exchange risk without delay." The reason is, of course, that larger positive or negative foreign exchange positions signify a larger exposure to foreign exchange risk, since sudden movements in the market can cause significant losses. This risk is higher the sharper are the fluctuations. In her discussion of the United States Foreign Exchange Market Revey (1981) notes that exchange rate volatility increased dramatically during the late seventies. This can be attributed not only to the adoption of the system of managed floating, but also to various disturbances in the world economy. It is then not surprising to read in Revey's report (1981, p. 38) that "the reluctance to carry exposures for even so short a period as overnight is underscored by data collected by

the United States Treasury showing a decline since 1977-78 in end-of-day positions for the most active trading banks."

So far I have discussed why a dealer generally wants to reduce his exposure to foreign exchange risk, but the connection to his optimal exchange rate remains to be explained. Recalling that the excess demand ED_i for a dealer's foreign exchange, as shown in (2.1), is negatively related to his own rate, it becomes clear that a dealer with a high foreign exchange position will tend to quote a low rate in order to induce his customers to buy from him. If, on the other hand, he is "short," he may try to entice the customers into selling foreign exchange to him by quoting a relatively high rate. The following quote by Tygier describes this type of dealer behavior.

> Let us assume that the dollar/mark is dealing in the market at a rate of 1.7355-65. A bank is called upon to make a market. If the bank does not wish to buy dollars, it will bid below the 1.7355 (we assume here that the spread made will be 10 points). If the bank wishes to sell dollars, it may make a market as aggressive as 46-56. (1983, p. 103)

Hence it seems reasonable to conclude that a dealer's optimal pricing policy should exhibit a negative relation between his inventory of foreign exchange and his optimal rate.

The second relation to be considered is that between a dealer's optimal exchange rate and his competitor's inventory. Assuming that the above conjecture holds, the competitor also will tend to quote a comparatively low rate when holding a relatively high positive inventory, while reacting to a negative inventory with a relatively high quote. Recalling that the excess demand ED_i for a dealer's foreign exchange is positively related to his competitor's exchange rate as

shown in (2.1), it becomes clear that the competitor's relatively high inventory will result in a relatively low excess demand for our dealer's foreign exchange, while this excess demand will be higher if his competitor has a lower, even negative, inventory. But the dealer will, of course, quote a higher rate the higher the expected excess demand for his foreign exchange. Hence I conclude that a dealer's optimal exchange rate should be negatively related to his competitor's currency holding.

Thus, looking at the foreign exchange market as a whole, all other things being equal, relatively high inventory holdings of all foreign exchange dealers will tend to be associated with relatively low individual dealer rates, and hence with a relatively low average market rate. But all other things are, of course, not equal. Particularly demand and supply for foreign exchange fluctuate dramatically from period to period. Hence it is rather the market's liquidity, which Tygier (1983, p. 101) defines as "the result of existing positions in the interbank market combined with arising supply and demand," that determines together with expectations the current market rate.

After having used my knowledge of the foreign exchange market to surmise that a dealer's optimal exchange rate should be negatively related to both his own and his competitor's inventory holdings, which also explains the behavior of the average market rate, I will now use the Model Without Interdealer Market presented above to derive analytically a dealer's optimal pricing strategy.

The Solution Technique

Although I have focused earlier on the ith dealer in discussing his decision-making represented by the value function $V^i(x_i,x_j)$, the

complete model includes two such value functions, $V^i(x_i,x_j)$ and $V^j(x_i,x_j)$, or, for simplicity, $V^1(x_1,x_2)$ and $V^2(x_1,x_2)$. They represent the fact that both foreign exchange dealers maximize their expected infinite-horizon profits by choosing their rates sequentially. Arguments by Bellman (1957) or Denardo (1967) can be used to guarantee that these value functions uniquely satisfy the infinite-horizon functional equation in (2.3) for i,j=1,2.

Two explicit assumptions concerning the relation between the first and the second dealer should be added. First I assume that both dealers decide simultaneously which rates to quote. Second the dealers neither discuss nor make any binding agreements concerning rates they will quote or strategies they will follow. This kind of behavior described in the latter assumption is commonly referred to as noncooperative behavior in the game theory literature (cf. Friedman, 1977; Intriligator, 1971).

Given all the above information, it is now possible to derive both dealers' optimal pricing strategies that constitute the Nash equilibrium of the system. This equilibrium is characterized by the fact that each dealer follows his own optimal pricing strategy, taking into account the other dealer's strategy. This implies that neither one of the dealers has an incentive to change his strategy, because he has already chosen that strategy that maximizes his expected infinite horizon profits under the assumption that his competitor does the same. The first dealer's optimal pricing strategy is derived explicitly in Appendix A. The second dealer's optimal pricing strategy, which can be derived analogously, is presented in the same appendix. As noted, a simultaneous solution procedure is used.

In the derivation, I first concentrate on the first dealer's decision-making process, which is characterized by (2.3), where i=1 and j=2. As mentioned earlier, it is assumed that this dealer knows both dealers' current inventories, the joint distribution $\Phi_{12}(\zeta_1,\zeta_2)$ of the random disturbances ζ_1 and ζ_2, and their respective marginal distributions. In addition, he is assumed to know the second dealer's optimal pricing strategy $p_2^*(x_1,x_2)$ of the form $p_2^*(x_1,x_2) = b_{21}x_1+b_{22}x_2+b_{23}$.

Following Bradfield and Zabel (1979) I hypothesize that the first dealer's value function $V^1(x_1,x_2)$ has the linear-quadratic form

$$(2.4) \qquad V^1(x_1,x_2) = \gamma_1 x_2^2 + \Pi_1 x_1 x_2 + \tfrac{1}{2}\delta_1 x_1^2 + \eta_2 x_2 + \Theta_1 x_1 + \rho_1,$$

which is also given by (A.3). As shown in Appendix A, this hypothesis can be used to replace the last term in (2.3) that represents the expected present value of all future profits. Now I am able to take the derivative of the transformed maximand in (2.3) and set it equal to zero in order to obtain the first dealer's optimal pricing strategy $p_1^*(x_1,x_2)$ as shown in (A.8), which is

$$(2.5) \qquad p_1^*(x_1,x_2) = -\frac{L_1 b_{21} - A_1 T_1 - \alpha\Pi_1 a_2}{R_1}\, x_1 - \frac{L_1 b_{22} - 2\alpha\gamma_1 a_2 - \alpha\Pi_1 A_1}{R_1}\, x_2 + b_{13}.$$

However, this expression still depends on the hypothetical coefficients of the second dealer's optimal pricing strategy. Thus an analogous derivation is followed in order to obtain the second dealer's optimal pricing strategy $p_2^*(x_1,x_2)$ which is given by (A.12), or

$$(2.6) \qquad p_2^*(x_1,x_2) = -\frac{L_2 b_{11} - 2\alpha\gamma_2 a_1 - \alpha\Pi_2 A_2}{R_2}\, x_1 - \frac{L_2 b_{12} - A_2 T_2 - \alpha\Pi_2 a_1}{R_2}\, x_2 + b_{23}.$$

Since (2.5) and (2.6) can be written as $p_1^*(x_1,x_2) = b_{11}x_1+b_{12}x_2+b_{13}$ and $p_2^*(x_1,x_2) = b_{21}x_1+b_{22}x_2+b_{23}$, equations (2.5) and (2.6) jointly determine b_{11}, b_{12}, b_{21}, and b_{22}. Hence the final form of the first dealer's optimal pricing strategy as shown in (A.16) is

$$\begin{aligned} p_1^*(x_1,x_2) &= \frac{(A_1T_1+\alpha\Pi_1a_2)R_2-L_1(2\alpha\gamma_2a_1+\alpha\Pi_2A_2)}{R_1R_2-L_1L_2}\,x_1 \\ &\quad + \frac{(2\alpha\gamma_1a_2+\alpha\Pi_1A_1)R_2-L_1(A_2T_2+\alpha\Pi_1a_1)}{R_1R_2-L_1L_2}\,x_2 + b_{13}. \end{aligned} \tag{2.7}$$

This optimal policy constitutes, together with an analogous policy for the second dealer represented in (A.17), the noncooperative Nash equilibrium of the system. Furthermore, it is shown in Appendix A that a system of six simultaneous equations, obtained through substitution of both optimal pricing pricing strategies and (2.4) into the original functional equation given by (2.3), jointly determines the coefficients γ_1, Π_1, δ_1, γ_2, Π_2, and δ_2 as follows:

$$\begin{aligned} \gamma_1 &= -\tfrac{1}{2}R_1b_{12}^2 + \tfrac{1}{2}(\alpha\delta_1-h_{c1})a_1^2b_{22}^2 + \alpha\gamma_1(1-A_2b_{22})^2 - \alpha\Pi_1a_1b_{22}(1-A_2b_{22}), \\ \Pi_1 &= -R_1b_{11}b_{12} - (\alpha\delta_1-h_{c1})(1-a_1b_{21})a_1b_{22} - 2\alpha\gamma_1A_2b_{21}(1-A_2b_{22}) \\ &\quad + \alpha\Pi_1((1-a_1b_{21})(1-A_2b_{22})+a_1A_2b_{21}b_{22}), \\ \delta_1 &= -R_1b_{11}^2 + (\alpha\delta_1-h_{c1})(1-a_1b_{21})^2 + 2\alpha\gamma_1A_2^2b_{21}^2 - 2\alpha\Pi_1(1-a_1b_{21})A_2b_{21}, \\ \gamma_1 &= -\tfrac{1}{2}R_1b_{21}^2 + \tfrac{1}{2}(\alpha\delta_2-h_{c2})a_2^2b_{11}^2 + \alpha\gamma_2(1-A_1b_{11})^2 - \alpha\Pi_2a_2b_{11}(1-A_1b_{11}), \\ \Pi_2 &= -R_2b_{22}b_{21} - (\alpha\delta_2-h_{c2})(1-a_2b_{12})a_2b_{11} - 2\alpha\gamma_2A_1b_{12}(1-A_1b_{11}) \\ &\quad + \alpha\Pi_2((1-a_2b_{12})(1-A_1b_{11})+a_2A_1b_{12}b_{11}), \\ \delta_2 &= -R_2b_{22}^2 + (\alpha\delta_2-h_{c2})(1-a_2b_{12})^2 + 2\alpha\gamma_2A_1^2b_{12}^2 - 2\alpha\Pi_2(1-a_2b_{12})A_1b_{12}. \end{aligned} \tag{2.8}$$

The above equations are identical to those given by (A.21) and (A.22).

Interpreting the Results

Using the dynamic programming framework, the Model Without Interdealer Market was formulated. It pictures how two foreign exchange dealers maximize their expected infinite-horizon profits by choosing their optimal exchange rates on the basis of both dealers' current inventories. At the beginning of each period in this infinite-horizon setting they decide simultaneously which rates to quote. Apart from the difference in starting inventories each dealer's decision-making process is identical from period to period. Therefore a time-invariant optimal pricing strategy was derived for each dealer that tells him for every possible combination of inventories which exchange rate to quote in order to maximize his expected infinite-horizon profits. The first dealer's optimal pricing strategy is shown in (2.7). Its coefficients b_{11}, b_{12}, and b_{13} depend on γ_1, Π_1, δ_1, γ_2, Π_2, and δ_2 which are jointly determined by (2.8). Due to the complexity of this simultaneous equation system, the coefficients of the dealer's optimal infinite-horizon pricing strategy have not yet been signed. Thus I have not been able to confirm the hypothesis that these coefficients are negative. A future plan is to use simulation techniques to analyze the solution of the equations in (2.8).

However, as a preliminary step, I have signed coefficients in special cases where the horizon is shortened. In particular, using the expressions derived for the infinite-horizon case, it was possible to derive an optimal pricing strategy for the cases of one and two periods in the horizon. Starting with the one-period-horizon model, the following functional equation, also shown in (A.23), represents the first dealer's decision-making process

$$V_1^1(x_1,x_2) = \max_{p_1} \{p_1(d_1+a_1p_2^*+A_1p_1) - \int H_1(x_1-d_1-a_1p_2^*-A_1p_1 -\zeta_1)d\Phi_1(\zeta_1)\}. \tag{2.9}$$

In (A.25) and (A.26) of Appendix A the first and second dealers' optimal pricing policies are then derived as

$$p_{11}^*(x_1,x_2) = -\frac{A_1A_2h_{c1}(2-A_2h_{c2})}{R_{11}R_{21}-L_{11}L_{21}}x_1 + \frac{a_1A_2h_{c2}(1-A_1h_{c1})}{R_{11}R_{21}-L_{11}L_{21}}x_2 + \frac{c_1}{R_{11}R_{21}-L_{11}L_{21}}, \tag{2.10}$$

and

$$p_{21}^*(x_1,x_2) = \frac{a_2A_1h_{c1}(1-A_2h_{c2})}{R_{11}R_{21}-L_{11}L_{21}}x_1 - \frac{A_1A_2h_{c2}(2-A_1h_{c1})}{R_{11}R_{21}-L_{11}L_{21}}x_2 + \frac{c_2}{R_{11}R_{21}-L_{11}L_{21}}, \tag{2.11}$$

respectively. Since $R_{11}R_{21}-L_{11}L_{21}=A_1A_2(2-A_1h_{c1})(2-A_2h_{c2})-a_1a_2(1-A_1h_{c1})(1-A_2h_{c2})>0$, all coefficients of x_1 and x_2 are negative. Thus I have obtained the expected result, namely that a dealer's optimal exchange rate is inversely related to both dealers' inventories.

If he has a long position, i.e., he holds a positive inventory, the dealer will tend to quote a lower rate than if he has a short position. The practice of quoting certain rates below or above the average rate in order to influence the customers towards purchasing or selling foreign exchange is called "shading". Kubarych notes the following on this subject:

> The trader knows the bank's present position in the currency in question. If the customer wants to buy sterling and the bank has a long position in sterling, the transaction

> would fit well. In that case, the trader may tend to shade the quotation to make it relatively more attractive to the customer. Conversely, if the bank has a short position in sterling, the trader may shade the quotation to make it relatively less attractive. (1983, p. 30)

Furthermore, if his competitor has a long position, and therefore will quote a relatively low price, our dealer will take into account the resulting decreased excess demand ED_i for his foreign exchange and will therefore tend to quote a relatively low rate, too.

However, a comparison of the coefficients of x_2 in $p^*_{11}(x_1,x_2)$ and $p^*_{21}(x_1,x_2)$ as given by (2.10) and (2.11), respectively, shows that a change in x_2 will always cause a larger change in p_2 than in p_1, or $|\partial p_2/\partial x_2|>|\partial p_1/\partial x_2|$. It means that a dealer's inventory has always a greater influence on his own exchange rate than on that of his competitor. This result is intuitive, since a dealer reacts directly to a change in his inventory with a change in his exchange rate, while his competitor only reacts to the change in his customers' expected excess demand caused by the first dealer's change in price.

Moreover, if the condition $|2A_i h_{ci}|>a_i h_{cj}$ is satisfied, a dealer's price will react more to a change in his own inventory than to an equal change of his competitor's inventory, or $|\partial p_i/\partial x_i|>|\partial p_i/\partial x_j|$. This is deduced from comparing the coefficients of x_1 and x_2 in $p^*_{11}(x_1,x_2)$, and analogously in $p^*_{21}(x_1,x_2)$. Since h_{ci} as shown in (2.2) is based on market interest rates for lending and borrowing, it is to be expected that market forces bring h_{c1} and h_{c2} close together, satisfying $|2A_i h_{ci}|>a_i h_{cj}$. However, the internal interest rate that the bank management charges the dealing department is not only a function of the lending and borrowing rates. It also depends on the management's

attitude towards risk. The higher h_{ci} the more risk averse is the bank, ceteris paribus. Hence it is conceivable, although rather unlikely, that for example the second bank is so risk averse, and h_{c2} so large, that the coefficient of x_2 in $p^*_{11}(x_1,x_2)$ exceeds that of x_1 in the same pricing strategy. Then a change in the second dealer's inventory will cause him to adjust his exchange rate so drastically that the first dealer will react to the resulting change in his customers' excess demand more than to an equal change of his own inventory. But this dominating effect of the competitor's inventory in a dealer's pricing strategy can only occur in one of the dealers' pricing policies, not in both at the same time, since $|2A_ih_{ci}|<a_ih_{cj}$ implies $|2A_jh_{cj}|>a_jh_{ci}$.

Furthermore, I conclude that the constant terms c_1 and c_2, as defined in (A.27) for both dealers' one-period-horizon pricing policies, are positive. This implies that the dealers will quote positive rates when they both have zero inventories. It can also be deduced from (A.27) that c_1 and c_2, and thus p^*_{11} and p^*_{21}, are greater the higher are $|A_i|$, d_i, h_{ci}, and $|h_{bi}|$. It means that higher excess demands and marginal holding costs (or interest rates) will cause the dealers to quote higher rates.

In order to verify that both dealers maximize rather than minimize their expected profits when following their optimal pricing strategies, it is shown in Appendix A that the second order conditions for profit maximization given by (A.28) and (A.29), or

$$(2.12) \qquad Dp_1p_1G^1_1 = A_1(2-A_1h_{c1})<0, \qquad Dp_2p_2G^2_1 = A_2(2-A_2h_{c2})<0$$

are satisifed.

Finally, substituting the optimal pricing strategies into (2.9), the coefficients of the one-period-horizon value function are derived as follows:

$$(2.13)\qquad \gamma_1 = -\frac{a_1^2 A_1 A_2^2 h_{c2}^2 (2-A_1 h_{c1})}{2(R_{11}R_{21}-L_{11}L_{21})^2}, \qquad \Pi_1 = -\frac{a_1 A_1 A_2 h_{c1} h_{c2} (2-A_1 h_{c1}) M_2}{(R_{11}R_{21}-L_{11}L_{21})^2},$$

$$\delta_1 = -\frac{h_{c1}(2A_1A_2(2-A_1h_{c1})(2-A_2h_{c2})M_2 + a_1^2 a_2^2 (1-A_2h_{c2})^2)}{(R_{11}R_{21}-L_{11}L_{21})^2},$$

which are also shown in (A.33). Since M_2 as defined by (A.32) is positive, I conclude that γ_1 is positive, and Π_1 and δ_1 are negative. Hence, the first dealer's one-period-horizon function $V_1^1(x_1,x_2)$ is concave in x_1 and convex in x_2, while $V_1^2(x_1,x_2)$ is convex in x_1 and concave in x_2.

In the model with two periods in the horizon it is also shown in Appendix A that both dealers' pricing strategies display a negative relationship between both dealers' inventories x_1 and x_2 and their optimal prices. This result again confirms my initial conjecture and is similar to that obtained in the one-period model. However, the computations are too complex to provide easy comparisons of relative signs of various coefficients. Hence, rather than analyzing the two-period model further, my plan is to defer additional analysis until an attempt is made to simulate the infinite-horizon equations in (2.8). While discussions of the one- and two-period models provide some confidence about signs of coefficients in the infinite horizon, verification of signs and other properties awaits analysis of these equations.

Before concluding this chapter let me make the following remarks. First I would like to stress that the foreign exchange dealers' optimal exchange rates quoted at the beginning of a particular period generally do not equalize the incoming supply and demand during that period, neither at an individual nor at an aggregate level. One reason is that supply and demand are partially random. Thus, assuming that it was in the dealers' interest that supply for foreign exchange equal demand, the best each dealer could do is to quote that rate at which expected excess demand would be equal to zero. But, more importantly, I need to address the question whether such a situation of zero excess demand would be desirable for the dealers. Due to uncertainty in the market the dealers do not have full control over their inventory positions, even though they are able to influence them through their quotes. Thus they generally do not hold inventories that coincide with their desired foreign exchange positions. At what levels these optimal positions would be has not yet been determined for the Model Without Interdealer Market. However, results by Bradfield and Zabel (1979) suggest that such optimal inventory levels exists. For their stock market model the authors derived an equilibrium inventory x^* that had the following two properties (1979, p. 64). "First . . . the specialist always adjusts expected inventory in the direction of x^*. Second . . . the long-run average level of inventory is x^*. In other words, x^* is the expected value of the stationary probability distribution of the specialist's inventory."

Hence, assuming that analogous equilibrium inventories exist in the foreign exchange market model towards which the dealers try to adjust their current inventory positions, I expect the dealers to quote those

rates that equalize their desired changes in inventory and their customers' expected excess demands. Only if the latter equal zero will the dealers quote prices at which expected demand equals expected supply. The above underlines an outstanding feature of any inventory model. The agent whose behavior is modeled is interested primarily in his inventory position and the implied present expected value of his profits, not in the question of whether or not markets clear. This kind of behavior does seem to be closer to reality than that of dealers, described in a typical one-period model without uncertainty, who trade only after a market clearing rate has been found by some anonymous pricing authority.

The second comment I would like to make concerns the assumption of stationarity in this model. The random terms of the excess demand facing the foreign exchange dealers are assumed to follow a stationary distribution. Following Bradfield and Zabel (1979) who found that distributions of their specialist's inventory and price were stationary, given the assumption of stationary random variables, I also expect that rates and inventory holdings of the dealers in the present model have a stationary distribution. Unfortunately, the assumption of stationarity is quite unrealistic as far as the foreign exchange market is concerned. First of all, the "market fundamentals" that determine the value of the foreign exchange rate in the long run change often. If, for example, interest rates drop in the country whose currency is traded, the current and future excess demand and the long-run value of this currency are affected. But, even when these market fundamentals remain constant, the expectations about future developments in the market might change. Einzig notes the following:

> Equilibrium rate is something essentially variable, fluid and elusive, and the extent of the adjustment is liable to change as and when the market's assessment of the situation changes, even in a complete absence of any change in the material facts, or their knowledge of the material facts, on which their assessment is based. (1966, p. 113)

Thus dealers in the real world foreign exchange market try to incorporate information concerning recent developments in the market into their decision-making process. In the present model, however, I assume that a dealer's expected excess demand as a function of dealer rates is unaffected by such information. Resulting changes in the actual excess demand are merely captured in the random term. Since the distribution of this random variable is stationary, the joint distribution of dealer rates also does not change over time. One justification for this simplification is that new information concerning the market never has the same impact on demand and supply for foreign exchange; thus it is very difficult to predict. To quote Tygier, who has traded in the foreign exchange market for many years,

> Anyone who has observed the foreign exchange markets for any time must have noted that there is a great difference between what the theoretical response of the markets to certain developments should be, and what actually takes place. In fact, the same news often has totally opposite effects. The focus of the markets changes constantly, and certain developments may at times be considered crucial, and at other times be totally ignored. (1983, p. 100)

In addition, it should be kept in mind that the object of this study is to focus on the dealers' trading behavior in order to explain the short run fluctuations of the foreign exchange rate around its long term value, not to actually explain how the latter is determined by macroeconomic variables.

CHAPTER III
THE MODEL WITH INTERDEALER MARKET

Introduction of a Dealer's Decision-Making Process

The Model Without Interdealer Market discussed in the previous chapter was based on the assumption that dealers trade only with their customers, not with each other. But in reality trading between dealers is a very important part of the foreign exchange market activity. In effect, it constitutes 86.6 percent of total trading in the United States Foreign Exchange Market, according to the Market Turnover Survey conducted in March 1986 by the Federal Reserve Bank of New York.

The benefit of interdealer trading is obvious. As Kubarych (1983, p. 34) puts it, "at any point there is the position traders have and the position they would like to have. They turn to the interbank market to harmonize the two." In other words, rather than just shading his own rate to encourage his customers to buy or sell foreign exchange, depending on the direction of his desired inventory change, a dealer can call another bank to initiate a trade which he will execute if he finds the bank's quote acceptable. Of course, in the two-agent framework used in this study, each dealer has only one counterpart with whom he can initiate a trade. Customer trading itself is one-sided, since the customers call upon the dealers at any time to obtain quotes at which they are free to trade, but they do not return this service to the dealers.

The introduction of interdealer trading into the foreign exchange market model provides each dealer with an additional decision variable

which henceforth will be called ordering, or z_i. This variable is defined such that $z_i>0$ implies that the ith dealer buys foreign exchange from the jth dealer at his rate p_j, while $z_i<0$ means that the ith dealer sells foreign currency to the jth dealer at p_j. Since both dealers make their ordering and pricing decisions simultaneously, a net trade of z_i-z_j results.

Apart from the incorporation of interdealer trading, the assumptions in the present model are identical to those made in Chapter II. The agents are again two foreign exchange dealers and their customers. As before, the ith dealer (where i=1,2) considers a horizon with an infinite number of trading days or periods of equal length. At the beginning of each period he faces the decisions of how much foreign exchange to trade actively with his competitor and which exchange rate to quote. He bases these decisions on his knowledge of both dealers' current inventories and his competitor's optimal pricing and ordering strategies. In addition, he takes into account the particular form of the excess demand function ED_i of his customers given by

$$(3.1) \qquad ED_i = d_i + a_i p_j + A_i p_i + \zeta_i,$$

where $i,j=1,2$ $(i\neq j)$, and d_i, a_i, and A_i are constants. As before, d_i and a_i are positive, A_i is negative, and $|A_i|$ is greater than a_i and a_j. The additive random variable ζ_i follows the joint distribution function $\Phi_{ij}(\zeta_i,\zeta_j)$ and the marginal distribution function $\Phi_i(\zeta_i)$ with expected value $E(\zeta_i)=0$, and finite variance $\sigma^2_{\zeta i}$ and covariance $\sigma^2_{\zeta i\zeta j}$.

Furthermore, the dealer considers his holding costs, which are, as in (2.2), given by

$$(3.2) \qquad H_i(w_i) = h_{ai} + h_{bi}w_i + h_{ci}\frac{w_i^2}{2},$$

where w_i represents the dealer's end-of-the-period inventory and h_{ai} and h_{ci} are positive constants, while h_{bi} is a negative constant.

The dealer's decision variables are his foreign exchange rate p_i and his ordering z_i. He chooses them simultaneously at the beginning of each period so that the expected present value of his infinite-horizon profits is maximized. Hence his decision-making process can be represented by the following dynamic programming functional equation:

$$V^i(x_i,x_j) = \max_{p_i,z_i} \{p_i(d_i+a_ip_j^*+A_ip_i+z_j^*) - p_j^*z_i - \int H_i(x_i+z_i-z_j^*$$

$$(3.3) \qquad -d_i-a_ip_j^*-A_ip_i-\zeta_i)d\Phi_i(\zeta_i) + \alpha \int V^i(x_i+z_i-z_j^*-d_i-a_ip_j^*$$

$$-A_ip_i-\zeta_i,x_j+z_j^*-z_i-d_j-a_jp_i-A_jp_j^*-\zeta_i)d\Phi_{ij}(\zeta_i,\zeta_j)\},$$

where $p_j^* = p_j^*(x_i,x_j)$ and $z_j^* = z_j^*(x_i,x_j)$ represent the jth dealer's optimal pricing and ordering strategies.

The interpretation of the above equation follows closely that of (2.3). The first two terms of the right-hand side of (3.3) depict the ith dealer's expected revenue in the current period. In particular, $p_i(d_i+a_ip_j^*+A_ip_i+z_j^*)$ stands for the dealer's revenue from his expected net sale (or purchase) initiated by the customers or the other dealer, while $-p_j^* z_i$ represents the dealer's revenue from actively buying (or selling) foreign exchange from his competitor. Both terms can be either positive or negative.

The third term represents, analogously to the second term in (2.3), the ith dealer's expected holding costs. The argument is in both cases his end-of-the-period inventory. However, in the present model this inventory is $w_i = x_i+z_i-z_j^*-ED_i$ as compared to $w_i = x_i-ED_i$ for the Model Without Interdealer Market. The difference $z_i-z_j^*$ between the two is the

change in the dealer's inventory through interdealer trading. Since the dealer does not know the realization of the random term at the beginning of the period, he makes his pricing and ordering decisions on the basis of the expected holding costs rather than the actual costs.

Finally, the fourth term represents the expected present value of the ith dealer's future infinite-horizon profits. It involves the value function $V^i(x_i,x_j)$ and incorporates the dealer's optimal sequential decision making. The discount rate $\alpha(0<\alpha<1)$ signifies that the dealer attributes a higher value to earlier returns than to later returns. The arguments of the value function V^i are both dealers' end-of-the-period inventories $w_i = x_i+z_i-z_j^*-ED_i$ and $w_j = x_j+z_j^*-z_i-ED_j$ which are also the dealers' starting inventories in the following period. It can be noted that the dealers' net trade $z_i-z_j^*$ enters these inventories with opposite signs. Since ED_i and ED_j are partially random, the ith dealer determines the expected value of his value function using his knowledge of the joint distribution fucntion $\Phi_{ij}(\zeta_i,\zeta_j)$. As mentioned in Chapter II, in order to compute this expectation I need to determine properties of the value function $V^i(x_i,x_j)$.

Since p_j^* and z_j^* are functions of x_i and x_j, the maximand in (3.3) depends on p_i, z_i, x_i, and x_j. But p_i and z_i are the ith dealer's decision variables. This means that he determines the expected present value of his current and future profits as a function of x_i and x_j for various choices of p_i and z_i and chooses for each combination of x_i and x_j that rate and that order which maximize this expected present value. With this behavior he actually follows a time-invariant optimal strategy that assigns to each combination of state variables (x_i,x_j) an optimal combination of decision variables (p_i,z_i).

The Optimal Pricing and Ordering Strategies

The Intuitive Result

As in Chapter II, I will begin the discussion of the dealer's optimal pricing and ordering strategies with a conjecture based on my knowledge of the real world foreign exchange market. Due to the inclusion of the interdealer market into the present model, the ith dealer's behavior is more complex than before, but it also conforms better with reality. Again the question arises how the dealer's optimal rate is related to both dealers' starting inventories. But, in addition, the link between a dealer's optimal ordering and inventory holdings needs to be discussed.

Starting with the connection between a dealer's optimal exchange rate p_i and his inventory x_i, I can expect an inverse relationship similar to that hypothesized for the dealer's optimal pricing strategy in the Model Without Interdealer Market. However, it should be kept in mind that the dealer now has two decision variables with which he can influence his inventory. Furthermore, his decision of how much foreign exchange to order (sell or buy) from his competitor has a more direct and predictable effect on his foreign exchange position than his pricing decision does. Therefore it is reasonable to assume that a dealer's response to a relatively high inventory with a relatively low exchange rate will be less pronounced in the Model With Interdealer Market than in the model discussed in the previous chapter.

Let me now focus on the relationship between a dealer's optimal ordering z_i and his inventory x_i. Recalling that a dealer tends to cover his foreign exchange position in order to avoid foreign exchange risk, assuming that he holds no definite view about future developments

in the market, I conjecture that he will tend to sell foreign exchange to his competitor if he carries a large positive inventory. Conversely, if he is short of foreign exchange, he can be expected to buy foreign currency from the other dealer. In other words, I hypothesize that the coefficient $\partial z_i^*/\partial x_i$ of the ith dealer's optimal ordering policy is negative. This kind of behavior is described by the textbook rule of covering discussed by Einzig (1966).

After having examined the dealer's price and ordering response to a change in his own inventory position, I need to concentrate on the relationship between his optimal rate p_i and ordering z_i and his competitor's inventory x_j. First it should be noted that the competitor is expected to display the same type of behavior as described above, thus responding to a relatively high inventory x_j with a relatively low rate p_j and a relatively low (here negative) ordering z_j, and conversely for a negative inventory x_j. But a relatively low p_j decreases the expected value of the excess demand ED_i facing the ith dealer and therefore increases the ith dealer's expected end-of-the-period inventory. Furthermore, a negative ordering z_j, or sale, of the jth dealer will also increase the ith dealer's inventory at the end of the period. Hence I expect that the dealer will lower his optimal price p_i, making the coefficient $\partial p_i^*/\partial x_j$ negative. This will increase the excess demand for his foreign exchange ED_i.

The dependence of the ith dealer's ordering decision z_i on the jth dealer's current inventory x_j is less clear-cut. His relatively high end-of-the-period inventory caused by his competitor's low p_j and z_j calls for a sale of foreign exchange, or a negative z_i. But since the competitor is quoting a low rate p_j, the first dealer's revenue for this

sale $-p_j z_i$ would be small, and so would be the cost $p_j z_i$ if he should decide to buy foreign exchange at this low rate instead of selling it. Whether this latter consideration dominates the earlier one, implying a positive sign of $\partial z_i^*/\partial x_j$, cannot be determined easily. The answer to this question may well depend on the relative sizes of the parameters involved in the model.

Looking at the market as a whole, I expect that both dealers' orders z_i and z_j will tend to offset each other if the dealers carry similar long or short positions. Within the framework of the present model this result can be partially explained by the fact that each dealer takes his competitor's optimal ordering strategy into account when making his own ordering decision. In terms of the real world foreign exchange market, the partial offsetting of the dealers' orders results from the fact that market-makers always have to be ready to quote a rate at which they will trade. If, for example, the ith dealer calls the jth dealer to sell z_i, he cannot in turn refuse to buy z_j at his rate p_i. If both dealers want to reduce their inventories, eventually they will have to lower their rates so far that the customers will buy more foreign exchange than sell. This again shows that the foreign exchange market's liquidity plays a decisive role in determining the average market rate. Tygier notes the following on this subject.

> Let us take the hypothetical situation where on balance the interbank market is very short of dollars, and has been for quite a while. If during the course of a day further demand for non-dollar currencies arises, the banks can easily accommodate this demand by reducing their short positions. This does not necessarily mean that the dollar will stabilize, because the banks may wish to re-establish their original positions. However, if a sudden need for dollars arises, we may have a situation where banks already overextended may

> have to buy large quantities of dollars that nobody has for sale. This would create a sharp--if temporary--disequilibrium. (1983, p. 101)

On the other hand, in the numerous cases where the dealers' inventories differ considerably from each other, the interdealer market works as a mechanism of exchange that brings the dealers' inventories closer to their desired positions. Therefore it reduces the role of the exchange rates to obtain these positions and hence exchange rate fluctuations.

Solving for the Optimal Strategies

The optimal pricing and ordering strategies for the Model With Interdealer Market are derived in Appendix B using the same solution technique as discussed in Chapter II. Again I assume that both dealers quote their rates and orders simultaneously and do not cooperate with each other with regard to rates and orders they choose or strategies they follow. Hence the set of both dealers' optimal strategies constitute the noncooperative Nash equilibrium of the system. This implies that there is no reason for either dealer to change his strategy, because each of them already follows the policy that maximizes the expected present value of his infinite-horizon profits, taking into account his competitor's optimal strategy.

The derivation of the optimal dealer strategies focuses on one dealer which will henceforth be called the first dealer. For i=1 and j=2, the dynamic programming functional equation in (3.3) represents this dealer's decision-making process. He is assumed to know his competitor's optimal pricing strategy $p_2^*(x_1,x_2) = b_{21}x_1+b_{22}x_2+b_{23}$ and ordering strategy $z_2^*(x_1,x_2) = s_{21}x_1+s_{22}x_2+s_{23}$, and the joint distribution function $\Phi_{12}(\zeta_1,\zeta_2)$ of the random variables ζ_1 and ζ_2.

As in Chapter II, I surmise that the first dealer's value function $V^1(x_1,x_2)$ characterized by (3.3) can be written as

$$V^1(x_1,x_2) = \gamma_1 x_2^2 + \Pi_1 x_1 x_2 + \tfrac{1}{2}\delta_1 x_1^2 + \eta_2 x_2 + \theta_1 x_1 + \rho_1, \tag{3.4}$$

or as in (B.4) of Appendix B.[1] Keeping in mind that the dealer in this model has two decision variables rather than one, it is noteworthy that the above form is identical to the hypothesized form for the Model Without Interdealer Market shown in (2.4).

In order to use (3.4) in (3.3), I replace the arguments x_1 and x_2 (the dealers' current starting inventories) of the value function V^1 in (3.4) with the dealers' starting inventories at the beginning of the next period $x_1+z_1-z_2^*-d_1-a_1p_2^*-A_1p_1-\zeta_1$ and $x_2+z_2^*-z_1-d_2-a_2p_1-A_2p_2^*-\zeta_2$. The resulting form can then be substituted into the fourth term of (3.3) to yield the functional equation given by (B.5), or

$$\begin{aligned} V^1(x_1,x_2) = \max_{p_1,z_1} \{ & p_1(d_1+a_1p_2^*+A_1p_1+z_2^*) - p_2^*z_1 - h_{a1} - h_{b1}(x_1+z_1-z_2^*-d_1-a_1p_2^* \\ & -A_1p_1) + \tfrac{1}{2}T_1(x_1+z_1-z_2^*-d_1-a_1p_2^*-A_1p_1)^2 + \tfrac{1}{2}T_1\sigma_{\zeta 1}^2 + \alpha\Pi_1(x_1+z_1-z_2^* \\ & -d_1-a_1p_2^*-A_1p_1)(x_2+z_2^*-z_1-d_2-a_2p_1-A_2p_2^*) + \alpha\Pi_1\sigma_{\zeta 1\zeta 2}^2 \\ & + \alpha\gamma_1(x_2+z_2-z_1-d_2-a_2p_1-A_2p_2^*)^2 + \alpha\gamma_1\sigma_{\zeta 2}^2 + \alpha\theta_1(x_1+z_1-z_2^*-d_1 \\ & -a_1p_2^*-A_1p_1) + \alpha\eta_1(x_2+z_2^*-z_1-d_2-a_2p_1-A_2p_2^*) + \alpha\rho_1 \}. \end{aligned} \tag{3.5}$$

As shown in Appendix B, after replacing p_2^* and z_2^* by $b_{21}x_1+b_{22}x_2+b_{23}$ and $s_{21}x_1+s_{22}x_2+s_{23}$, respectively, I derive the first dealer's optimal pricing and ordering strategies by taking the derivatives of the above

[1] To economize on notation, the symbols of the Model Without Interdealer Market are used in the present model for corresponding coefficients. However, the values of these coefficients are not identical in the two models.

maximand with respect to p_1 and z_1 and setting them equal to zero. Solving the two equations simultaneously for p_1 and z_1 yields the coefficients of the first dealer's optimal pricing and ordering strategies p_1^* and z_1^* as functions of b_{21}, s_{21}, and other coefficients as given by

$$(3.6) \quad \begin{aligned} b_{11} &= \frac{1}{D_1}[Q_{1b}b_{21}+Q_{1s}s_{21}+Q_{11}], & b_{12} &= \frac{1}{D_1}[Q_{1b}b_{22}+Q_{1s}s_{22}+Q_{11}], \\ s_{11} &= \frac{1}{D_1}[Z_{1b}b_{21}+Z_{1s}s_{21}+Q_{12}], & s_{12} &= \frac{1}{D_1}[Z_{1b}b_{22}+Z_{1s}s_{22}+Q_{13}]. \end{aligned}$$

Equations (3.6) are also shown in (B.15), and (B.16) defines other coefficients. With an appropriate interchange of subscripts, an analogous system of equations can be derived for the second dealer's coefficients as given by (B.17). The two systems jointly determine the coefficients for both dealers. Hence I simultaneously solve for these coefficients to derive the first dealer's optimal pricing strategy as

$$(3.7) \quad p_1^*(x_1,x_2) = \frac{R_{11}N_{11}+N_{12}J_{12}}{J_{11}J_{12}-R_{11}R_{12}}x_1 + \frac{R_{11}M_{11}+M_{12}J_{12}}{J_{11}J_{12}-R_{11}R_{12}}x_2 + b_{13},$$

and his optimal ordering strategy as

$$(3.8) \quad p_1^*(x_1,x_2) = \frac{J_{11}N_{11}+N_{12}R_{12}}{J_{11}J_{12}-R_{11}R_{12}}x_1 + \frac{R_{12}M_{12}+M_{11}J_{11}}{J_{11}J_{12}-R_{11}R_{12}}x_2 + s_{13},$$

with R_{11}, N_{11}, etc. defined by (B.19) and (B.22). The second dealer's optimal strategies are analogous. Together, the two dealers' strategies constitute the noncooperative Nash equilibrium of the system. Neither dealer can increase the present expected value of his infinite-horizon profits by following another strategy.

In Appendix B these optimal dealer strategies are substituted back into (3.5) and the resulting equation has the form shown in (3.4) which confirms this hypothesis. Furthermore, equating corresponding parameters, I obtain a system of six equations (for both dealers) that jointly determine the coefficients γ_1, Π_1, δ_1, γ_2, Π_2, and δ_2 as shown in (B.27).

Interpreting the Results

The Model With Interdealer Market discussed in this chapter distinguishes itself from the Model Without Interdealer Market by the additional choice variable z_i that each dealer has at his disposition. This implies that the noncooperative Nash equilibrium for the present model consists of a set of two optimal strategies for each dealer rather than just one pricing strategy per dealer. Nevertheless, the value functions $V^i(x_i,x_j)$ in the two models have the same functional form, which is shown in (2.4) and (3.4). Therefore it is not surprising that the same type of complications are encountered as in Chapter II when trying to determine the signs of the coefficients of the optimal strategies given by (3.7) and (3.8). As before, these coefficients depend on the six coefficients γ_1, Π_1, δ_1, γ_2, Π_2, and δ_2, which are jointly determined by six equations. Three of these equations are represented in (B.27) of Appendix B, the remaining three can be obtained by interchanging appropriate subscripts. Since this simultaneous equation system is even more complex than that for the Model Without Interdealer Market, the coefficients of the dealer's optimal infinite-horizon pricing and ordering strategies have not yet been signed. Therefore the conjecture concerning the signs of these coefficients cannot be

confirmed at this point. Its confirmation awaits further analysis of the equations in (B.27), possibly with the use of simulation techniques.

However, as a first approach, the special case of one period in the horizon has been examined and the coefficients of the dealer's optimal pricing and ordering strategies have been signed. The dealer's decision-making process in the one-period-horizon model is represented by (B.28) in Appendix B, or by

$$V_1^1(x_1,x_2) = \max_{p_1,z_1} \{p_1(d_1+a_1p_2^*+A_1p_1+z_2^*) - p_2^*z_1 - \int H_1(x_1+z_1 -z_2^*-d_1-a_1p_2^*-A_1p_1-\zeta_1)d\Phi_1(\zeta_1)\}. \tag{3.9}$$

This functional equation is identical to the infinite-horizon functional equation shown in (3.3) except for the fact that the latter consists of one additional term that represents the expected present value of all future profits. The first dealer's optimal pricing and ordering strategies that are implied by (3.9) are derived in Appendix B and given by

$$p_{11}^*(x_1,x_2) = \frac{4A_1A_2h_{c1}^2h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}} \{(A_1+A_2-a_1)x_1 + (2A_2+A_1-\frac{1}{h_{c2}})x_2\} + b_{13}, \tag{3.10}$$

and

$$z_{11}^*(x_1,x_2) = -\frac{4A_1A_2[3A_1A_2+A_2^2+A_1a_2+A_2a_2-a_1a_2-\frac{2A_2}{h_{c2}}]h_{c1}^2h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}}x_2 - \frac{4A_1A_2[A_1(A_2+a_2)h_{c1} + (A_2-a_2)]h_{c1}h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}}x_2 + s_{13}, \tag{3.11}$$

respectively, where $J_{11}J_{12}-R_{11}R_{12}>0$ is defined by (B.34). Equations (3.10) and (3.11) are also shown in (B.35) and (B.36). It should be noted that the second order conditions for profit maximization are

satisfied, as shown in (B.37) of Appendix B. Furthermore, if (3.10) and (3.11) are substituted into (3.9), I obtain a value function of the form shown in (3.4) with coefficients δ_1, Π_1, and γ_1 as derived in (B.40) of the same appendix. It can be concluded from (B.40) that δ_1 and Π_1 are negative, while γ_1 is positive. The same result had been obtained for the one-period-horizon Model Without Interdealer Market.

Let me first examine how a dealer's pricing and ordering decisions are related to his own inventory position. From (3.10) it follows that $\partial p^*_{11}/\partial x_1$ is negative, thus confirming my conjecture that a dealer's optimal exchange rate is inversely related to his inventory. Furthermore, (3.11) shows that $\partial z^*_{11}/\partial x_1$ is negative. This result also confirms that a dealer's ordering is inversely related to his inventory position for this one-period-horizon case. An example for the dealer's behavior described above is given in the following quotation from Coninx. However, it should be noted that he quotes the exchange rates in European terms, rather than in the direct terms used in this study, which implies that the inverse relationship between the dealer's rate and his inventory appears to be direct.

> The market-maker in A now has a short position in Cbs [foreign currency units] and as the currencies have been fairly stable for a long time and it is early in the dealing day, he may adjust his quotation to reflect his interest in buying Cbs to cover his short position . . . . Assuming he would like to cover the exposure, he can either buy back the Cbs in the open market at 1.9998 or hope that a natural seller will show up at better than 1.9999. To encourage sellers he will no doubt make it interesting for them and improve on the market and quote 1.99975 - 1.99995, marginally better than the market's buying price of 2.0000. (1980, p. 94)

As mentioned earlier, it is to be expected that the inclusion of the choice variable z_i affects the dependence of the dealer's optimal

rate p_i^* on his inventory position x_i. Since the ith dealer can now actively alter his position by trading with his competitor, he will shade his price (in order to encourage his customers to trade with him in the desired direction) to a lesser extent than in the Model Without Interdealer Market. In terms of the pricing strategies, I therefore expect that the coefficient $\partial p_i^*/\partial x_i$ in the Model With Interdealer Market is absolutely smaller than the same coefficient in the Model Without Interdealer Market. However, verification of this conjecture awaits further analysis of both models with the use of simulation techniques.

Focusing on $\partial p_{11}^*/\partial x_2$, the dependence of the first dealer's optimal rate on his competitor's inventory position as shown in (3.10), I am able to conclude, as expected, that this coefficient is negative for the one-period-horizon case. Thus the higher the competitor's inventory x_2, the lower are his optimal rate p_2^* and ordering z_2^*, and the higher is the dealer's expected end-of-the-period inventory $x_1+z_1-z_2^*-d_1-a_1p_2^*-A_1p_1$. To counterbalance this effect, the ith dealer will choose a lower rate p_1^*.

However, a comparison of $\partial p_{11}^*/\partial x_1$ and $\partial p_{11}^*/\partial x_2$ yields the somewhat surprising result that the latter is absolutely greater than the former. This is contrary to the result for the Model Without Interdealer Market, where $|\partial p_{11}^*/\partial x_1|$ was greater than $|\partial p_{11}^*/\partial x_2|$. Without doubt, this change in relative sizes must be due to the inclusion of the additional decision variable z_i. First, as discussed earlier, the fact that the first dealer can now change his inventory through z_1 reduces his price dependence of his inventory, or $|\partial p_{11}^*/\partial x_1|$. Second, although an increase (decrease) of x_2 increases (decreases) the first dealer's expected end-of-the-period inventory through changes in p_2^* and z_2^*, it is not clear what the first dealer's ordering response will be. If I only

consider his expected inventory, he should order less (more). On the other hand, taking into account his competitor's low (high) exchange rate, he might want to order more (less). Which effect dominates, and hence which sign $\partial z_{11}^{*}/\partial x_2$ has, cannot be determined from (3.11). But it is conceivable that the first dealer predominantly resorts to his rate (as opposed to his ordering) to respond to changes in his competitor's inventory. These two arguments offer some rationale for the result that $|\partial p_{11}^{*}/\partial x_1| < |\partial p_{11}^{*}/\partial x_2|$ in the Model With Interdealer Market. It should therefore be kept in mind that the dealer's pricing strategy in the present model is only part of his overall dealer strategy. Hence the dealer's pricing behavior may appear counterintuitive if viewed independently, but make perfect sense if considered together with his ordering behavior.

Nevertheless, comparing the two coefficients of the first dealer's optimal ordering strategy $z_{11}^{*}(x_1,x_2)$ as shown in (3.11), it follows that s_{11}, the coefficient of x_1, is smaller than s_{12}, the coefficient of x_2. This allows for the possibility that s_{12} may be either negative or positive, while s_{11} is clearly negative. The result implies that an increase (decrease) in a dealer's inventory causes him to sell (buy) more foreign exchange than an equal increase in his competitor's inventory would.

Another interesting conclusion can be drawn from the comparison of both dealers' ordering policies. The second dealer's ordering policy is, analogously to (3.11), given by

$$z_{21}^{*}(x_1,x_2) = -\frac{4A_1A_2[3A_1A_2+A_1^2+A_1a_2+A_2a_1-A_1a_1-a_1a_2-\frac{2A_1}{h_{c1}}]h_{c1}^2h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}}\,x_2$$

$$- \frac{4A_1A_2[A_2(A_1+a_1)h_{c2} + (A_1-a_1)]h_{c1}^2h_{c2}}{J_{11}J_{12}-R_{11}R_{12}} x_1 + s_{23}. \tag{3.12}$$

If I now consider an increase in x_1 by one unit, the first dealer's reaction is clearly to reduce z_1, i.e., to buy less or sell more. The second dealer's reaction, on the other hand, is less clear-cut. He wants to reduce his ordering z_2 by $4A_1A_2^2(A_1+a_2)h_{c1}^2h_{c2}^2/(J_{11}J_{12}-R_{11}R_{12})$ and increase it by $-4A_1A_2(A_1-a_1)h_{c1}^2h_{c2}/(J_{11}J_{12}-R_{11}R_{12})$. But the reduction of z_2 is dominated by the reduction in z_1. Hence the change in net trade $\partial z_{11}^*/\partial x_1 - \partial z_{21}^*/\partial x_1$ is negative, where the net trade, or net sale, is defined as the difference z_1-z_2 between the first and the second dealer's ordering. Since the change of net trade is negative, I conclude that an increase in the first dealer's inventory causes an increase in the net sale of foreign exchange from the first dealer to the second, while a decrease in his inventory results in an increase in the first dealer's net purchase, or a decrease in his net sale. Of course, analogous conclusions can be drawn for a change in the second dealer's inventory. This result implies that a dealer's change in inventory is always countered by a net trade between the dealers in the opposite direction. Furthermore, the amount of net trade resulting from a unit change in the ith dealer's inventory, for i,j=1,2 ($i\neq j$), is $-4A_1A_2[2A_1A_2+A_j^2+A_ia_j+A_ja_j-a_1a_2-(2A_j+A_i-a_i)/h_{cj}]h_{c1}^2h_{c2}^2/(J_{11}J_{12}-R_{11}R_{12})$. Using (B.34), it can be noted that if the condition $(A_1A_2+A_i^2+A_ia_i +A_ja_i)h_{c1}h_{c2}-(2A_i+A_j)h_{cj}+1 > a_ih_{ci}$ is satisfied, which is to be expected, since h_{c1} and h_{c2} are driven together by a competitive money market, then the net trade induced by a dealer's inventory change is smaller than the initial change.

The above discussion underlines the function of the interdealer market. It is to give the banks, which are, according to Wasserman, Prindl, and Townsend (1972), "profit-seeking and risk-bearing institutions," which "accept risks in the process of maximizing returns to their shareholders," the possibility to cover at least part of their foreign exchange exposure. In other words, the banks, which serve their customers as risk-bearers, resort to the interdealer market to reduce their own risk by equalizing their individual inventories. The real world interdealer market is, of course, more advantageous for the dealers than a two-agent market, because it offers more than one counterpart to trade with. Coninx notes

> Whether intensive inter-bank dealings bring great benefit to the participating banks is questionable; nevertheless it helps to create a wide coverage of maturities and currencies that would otherwise not exist. As these risks are spread over many banks--in places like London a hundred or more--there is little possibility that any loss arising would be so large as to have a detrimental effect upon one bank. (1980, p. 46)

But let me return to the two-agent model to analyze briefly the special case where two dealers are identical in the following sense. First they have the same holding cost functions, and second their customers' expected excess demands are identical. In addition, the random variables ζ_1 and ζ_2 follow the same marginal distributions, while their realizations are in general not equal. The latter implies that the dealers' inventory positions generally differ. Without doubt, in this special case, the first and second dealer's optimal pricing and ordering strategies will be identical. This means that the first dealer's optimal rate p_{11}^* and ordering z_{11}^* for given inventory positions x_1 and x_2 equals the second dealer's optimal rate p_{21}^* and ordering z_{21}^*

for the case where the dealers' inventory positions are reversed, no matter in which period these inventories are held. Furthermore, assuming that there are equilibrium inventories x_1^* and x_2^* (cf. Bradfield and Zabel, 1979), such that the dealers' optimal price and ordering response to those inventories would yield expected end-of-the-period inventories identical to x_1^* and x_2^*, then for this special case x_1^* equals x_2^*. Of course, the equilibrium prices $p_{i1}^*(x_1^*,x_2^*)$ and equilibrium orders $z_{i1}^*(x_1^*,x_2^*)$ would also be the same for i=1 and i=2, which implies a zero net trade for this particular equilibrium trade position.

Let me now assume that at the beginning of the period the first dealer holds an inventory $x_1=x_1^*+1$, while the second dealer holds an inventory $x_2=x_2^*=x_1^*$. Using the results derived earlier, I can conclude that the first dealer's inventory which is one unit above his equilibrium inventory will result in a net sale $|z_1-z_2|$ of foreign exchange from the first to the second dealer. This will decrease the first dealer's exposure to risk, while increasing that of the second dealer, which is nothing else but the spreading of risk over more than one dealer. Furthermore, it can be seen that both dealers' optimal rates $p_{11}^*(x_1^*+1,x_2^*)$ and $p_{21}^*(x_1^*+1,x_2^*)$ will lie below the dealers' equilibrium rates. This shows that a random shock affecting one dealer's inventory in one period is transmitted through the market mechanism so that it affects both dealers' exchange rates in the next period.

However, to quote Hudson (1979, p.ix), "it should be remembered that no two banks or customers react in exactly the same way or have necessarily the same interests." Therefore the above conclusions for the behavior of identical dealers would have to be modified to take the differences in the dealers' customer structure and holding cost (which includes the banks' different attitude towards risk) into account.

CHAPTER IV
CONCLUSION

In the present study I have attempted to describe the microstructure of the foreign exchange market. This market is an over-the-counter market in which dealers constantly trade foreign exchange with their customers and with each other. There is no central authority that calls out a single market rate at which trades take place and markets clear. Instead, the dealers make individual pricing and trading decisions on the basis of their inventory holdings and expectations about future developments in the market.

To obtain some insight into this ongoing trading activity and the determination of exchange rates, I have focused on the decision-making process of typical foreign exchange dealers. In this process each dealer, who is exposed to randomness in both the customer and the interdealers markets, follows an optimal pricing and trading strategy, taking into account that his competitors behave in a similar fashion. Two infinite-horizon dynamic programming models have been developed, which consider the operation of a two-dealer market.

The first is called the Model Without Interdealer Market. In this model, the two dealers exchange foreign currency with their customers, but they do not trade with each other. Nevertheless, the two dealers' activities are interrelated, since each dealer's exchange rate affects the excess demand for the other dealer's foreign exchange. In the infinite-horizon framework both dealers' time-invariant, optimal pricing

strategies have been derived. They are based on the assumption that each dealer maximizes the expected present value of his infinite-horizon profits by choosing his rates sequentially and that he takes into account his competitor's optimal strategy when doing so. Thus the two optimal pricing strategies constitute the noncooperative Nash equilibrium of the two-dealer market.

While the exact Nash equilibrium has been derived in a simultaneous solution procedure, assuming a linear-quadratic framework, the complexity of the solution makes it difficult to provide a complete interpretation of the outcomes. Hence Nash equilibria for finite versions have also been developed to assist in interpretation. I was able to conclude for the special cases of one and two periods in the horizon that a dealer's optimal exchange rate for a given period is negatively related to both dealers' inventory positions at the beginning of that particular period. This result is intuitive and it reconfirms verbal descriptions regarding the pricing behavior of foreign exchange dealers. It explains the dealers' practice of covering their foreign exchange positions and the inverse movement of individual dealer and average market rates in response to liquidity changes in the foreign exchange market.

Subsequently, the Model Without Interdealer Market has been developed to allow for the fact that dealers do not only trade with their customers, but also with each other. This second model represents the real world foreign exchange market more closely, since the bulk of actual foreign exchange activity consists of interdealer trading. In addition to the exchange rate, each dealer now has a second decision variable called ordering. It means that he determines the amount of

foreign exchange to trade with the other dealer at the same time as choosing his own exchange rate. That decision-making process is also repeated in every period. As before, and using similar solution techniques, a Nash equilibrium has been derived for this model. It consists of the two optimal pricing strategies and, in addition, of two optimal ordering strategies.

As in the previous model, due to the complexity of the solution, a one-period-horizon version of the Model With Interdealer Market has been presented to assist in interpreting outcomes. Each dealer's optimal pricing strategy in this one-period-horizon model displays a similar inverse relationship between a dealer's optimal rate and both dealers' starting inventories for a given period as observed in the Model Without Interdealer Market. Furthermore, it could be shown that a dealer's optimal ordering (sale or purchase) of foreign exchange from his competitor is negatively related to the dealer's inventory. This confirmed the conjecture that a dealer uses his ordering in the interdealer market to exercise a direct and predictable influence on his inventory in direction of the desired inventory position, as opposed to an indirect and partially uncertain influence exercised by the dealer's optimal rate. Although I could not determine the sign of the relationship between a dealer's optimal ordering and his competitor's inventory without making explicit assumptions about the relative sizes of involved parameters, it could be shown that a change in a dealer's inventory results in a net trade between the two dealers that is opposite to and smaller than the original inventory change. In verbal descriptions of the foreign exchange market this behavior is referred to as the function of the interdealer market, namely to allow the dealers to cover their positions and spread their risk.

Plans for future research include the simulation of the simultaneous equation systems that have been derived for the undetermined coefficients in the two infinite-horizon models. This will assist me in interpreting the foreign exchange dealers' optimal infinite-horizon strategies for the Models With and Without Interdealer Market. In particular, I expect to be able to show that these strategies display the same type of behavior as described by the finite-horizon strategies examined in this study. It will also enable me to examine the equilibrium values for the dealers' expected inventories and the adjustment properties of the system towards these equilibrium values. Furthermore, the analysis of the variance of the dealers' exchange rates should yield the following results. First I expect that the intuitive assumption of a positive correlation between the two dealers' random variables will result in a positive correlation between these dealers' optimal rates. Second it should be possible to conclude from a comparison of the Models With and Without Interdealer Market that interdealer trading has a dampening effect on exchange rate fluctuations since it allows the dealers to adjust their inventory holdings other than by just their prices.

APPENDIX A

DERIVATION OF AN OPTIMAL PRICING STRATEGY FOR THE MODEL WITHOUT INTERDEALER MARKET

The Infinite-Horizon Model

Following Bradfield and Zabel (1979), the optimal pricing strategy of the first dealer will be derived here, assuming that he takes as given his competitor's optimal pricing strategy $p_2^*(x_1,x_2)$ which is hypothesized to be of the form

$$p_2^*(x_1,x_2) = b_{21}x_1 + b_{22}x_2 + b_{23}. \tag{A.1}$$

In addition, the first dealer is assumed to know both dealers' inventories x_1 and x_2, the joint distribution $\Phi_{12}(\zeta_1,\zeta_2)$, and the marginal distributions $\Phi_1(\zeta_1)$ and $\Phi_2(\zeta_2)$ of the random variables ζ_1 and ζ_2. The first dealer's functional equation is then given by

$$\begin{aligned} V^1(x_1,x_2) = \max_{p_1} \{p_1(d_1+a_1p_2^*+A_1p_1) - \int H_1(x_1-d_1-a_1p_2^* \\ - A_1p_1-\zeta_1)d\Phi_1(\zeta_1) + \alpha \int V^1(x_1-d_1-a_1p_2^*-A_1p_1 \\ - \zeta_1, x_2-d_2-a_2p_1-A_2p_2^*-\zeta_2)d\Phi_{12}(\zeta_1,\zeta_2)\}. \end{aligned} \tag{A.2}$$

It will be hypothesized that the first dealer's value function is of the form

$$V^1(x_1,x_2) = \gamma_1x_2^2 + \Pi_1x_1x_2 + \tfrac{1}{2}\delta_1x_1^2 + \eta_1x_2 + \Theta_1x_1 + \rho_1. \tag{A.3}$$

Replacing the next period's value function in (A.2) by the above hypothesized form and substituting (2.2) into (A.2), I can write

$$V^1(x_1,x_2) = \max_{p_1} \{p_1(d_1+a_1p_2^*+A_1p_1) - h_{a1} - h_{b1}(x_1-d_1-a_1p_2^*-A_1p_1)$$

$$
\text{(A.4)} \quad \begin{aligned}[t]
&+ \tfrac{1}{2}T_1(x_1-d_1-a_1p_2^*-A_1p_1)^2 + \tfrac{1}{2}T_1\sigma_{\zeta 1}^2 + \alpha\gamma_1(x_2-d_2-a_2p_1-A_2p_2^*)^2 \\
&+ \alpha\gamma_1\sigma_{\zeta 2}^2 + \alpha\Pi_1(x_1-d_1-a_1p_2^*-A_1p_1)(x_2-d_2-a_2p_1-A_2p_2^*) + \alpha\Pi_1\sigma_{\zeta 1\zeta 2}^2 \\
&+ \alpha\eta_1(x_2-d_2-a_2p_1-A_2p_2^*) + \alpha\theta_1(x_1-d_1-a_1p_2^*-A_1p_1) + \alpha\rho_1\},
\end{aligned}
$$

where $T_1 = \alpha\delta_1 - h_{c1}$. Substitution of (A.1) into (A.4) yields, after rearranging terms,

$$
\begin{aligned}
V^1(x_1,x_2) = \max_{p_1} \{ & [\tfrac{1}{2}A_1(2+A_1T_1) + \alpha\gamma_1a_2^2 + \alpha\Pi_1a_2A_1]p_1^2 + [([a_1(1+A_1T_1) \\
& + 2\alpha\gamma_1a_2A_2 + \alpha\Pi_1(a_1a_2+A_1A_2)]b_{21} - A_1T_1 - \alpha\Pi_1a_2)x_1 \\
& + ([a_1(1+A_1T_1) + 2\alpha\gamma_1a_2A_2 + \alpha\Pi_1(a_1a_2+A_1A_2)]b_{22} - 2\alpha\gamma_1a_2 \\
& - \alpha\Pi_1A_1)x_2 + (1+A_1T_1 + \alpha\Pi_1a_2)(d_1+a_1b_{23}) + (2\alpha\gamma_1a_2 \\
\text{(A.5)} \qquad & + \alpha\Pi_1A_1)(d_2+A_2b_{23}) + A_1h_{b1} - \alpha\theta_1A_1 - \alpha\eta_1a_2]p_1 \\
& + [\tfrac{1}{2}T_1(1-a_1b_{21})^2 + \alpha\gamma_1A_2^2b_{21}^2 - \alpha\Pi_1(1-a_1b_{21})A_2b_{21}]x_1^2 \\
& - [T_1(1-a_1b_{21})a_1b_{22} + 2\alpha\gamma_1A_2b_{21}(1-A_2b_{22}) - \alpha\Pi_1((1-a_1b_{21}) \\
& (1-A_2b_{22}) - a_1b_{22}A_2b_{21})]x_1x_2 + [\tfrac{1}{2}T_1a_1^2b_{22}^2 + \alpha\gamma_1(1-A_2b_{22})^2 \\
& - \alpha\Pi_1a_1b_{22}(1-A_2b_{22})]x_2^2 + \text{const. } x_1 + \text{const. } x_2 + \text{const.}\}.
\end{aligned}
$$

The first derivative with respect to p_1 of the maximand $G^1(x_1,x_2,p_1)$ in (A.5) is then

$$
\begin{aligned}
Dp_1G^1 = & [A_1(2+A_1T_1) + 2\alpha\gamma_1a_2^2 + 2\alpha\Pi_1a_2A_1]p_1 + ([a_1(1+A_1T_1) + 2\alpha\gamma_2a_2A_2 \\
\text{(A.6)} \qquad & + \alpha\Pi_1(a_1a_2+A_1A_2)]b_{21} - A_1T_1 - \alpha\Pi_1a_2)x_1 + ([a_1(1+A_1T_1) + 2\alpha\gamma_2a_2A_2 \\
& + \alpha\Pi_1(a_1a_2+A_1A_2)]b_{22} - 2\alpha\gamma_1a_2 - \alpha\Pi_1A_1)x_2 + (1+A_1T_1+\alpha\Pi_1a_2) \\
& (d_1+a_1b_{23}) + (2\alpha\gamma_1a_2+\alpha\Pi_1A_1)(d_2+A_2b_{23}) + A_1h_{b1} - \alpha\theta_1A_1 - \alpha\eta_1a_2,
\end{aligned}
$$

and the second derivative with respect to p_1 is

$$
\text{(A.7)} \qquad Dp_1p_1G^1 = A_1(2+A_1T_1) + 2\alpha\gamma_1a_2^2 + 2\alpha\Pi_1a_2A_1,
$$

which has to be negative to guarantee that the first dealer maximizes his expected infinite horizon profits when he follows his optimal

pricing strategy. This optimal strategy $p_1^*(x_1,x_2)$ is obtained by setting the first derivative equal to zero, and it can be written as

$$\text{(A.8)}\qquad p_1^*(x_1,x_2) = -\frac{L_1b_{21}-A_1T_1-\alpha\Pi_1a_2}{R_1}x_1 - \frac{L_1b_{22}-2\alpha\gamma_1a_2-\alpha\pi_1A_1}{R_1}x_2 + b_{13},$$

or

$$\text{(A.9)}\qquad p_1^*(x_1,x_2) = b_{11}x_1 + b_{12}x_2 + b_{13},$$

where

$$b_{11} = -\frac{L_1b_{21}-A_1T_1-\alpha\Pi_1a_2}{R_1}, \quad b_{12} = -\frac{L_1b_{22}-2\alpha\gamma_1a_2-\alpha\pi_1A_1}{R_1},$$

$$\text{(A.10)}\qquad b_{13} = -\frac{1}{R_1}[(1+A_1T_1+\alpha\Pi_1a_2)(d_1+a_1b_{23}) + (2\alpha\gamma_1a_2+\alpha\Pi_1A_1)(d_2 + A_2b_{23}) + A_1h_{b1} - \alpha\theta_1A_1 - \alpha\eta_1a_2],$$

and

$$\text{(A.11)}\qquad \begin{aligned} L_1 &= a_1(1+A_1T_1) + 2\alpha\gamma_1a_2A_2 + \alpha\Pi_1(a_1a_2+A_1A_2), \\ R_1 &= A_1(2+A_1T_1) + 2\alpha\gamma_1a_2^2 + 2\alpha\Pi_1a_2A_1. \end{aligned}$$

Following an analogous derivation procedure to that shown in (A.1) through (A.8), focusing now on the second dealer, his optimal pricing strategy $p_2^*(x_1,x_2)$ can be derived as follows:

$$\text{(A.12)}\qquad p_2^*(x_1,x_2) = -\frac{L_2b_{11}-2\alpha\gamma_2a_1-\alpha\Pi_2A_2}{R_2}x_1 - \frac{L_2b_{12}-A_2T_2-\alpha\Pi_2a_1}{R_2}x_2 + b_{23},$$

implying

$$b_{21} = -\frac{L_2b_{11}-2\alpha\gamma_2a_1-\alpha\Pi_2A_2}{R_2}, \quad b_{22} = -\frac{L_2b_{12}-A_2T_2-\alpha\Pi_2a_1}{R_2},$$

$$\text{(A.13)}\qquad b_{23} = -\frac{1}{R_2}[(1+A_2T_2+\alpha\Pi_2a_1)(d_2+a_2b_{13}) + (2\alpha\gamma_2a_1+\alpha\Pi_2A_2)(d_1 +A_1b_{13}) + A_2h_{b2} - \alpha\theta_2A_2 - \alpha\eta_2a_1],$$

and

$$\text{(A.14)} \qquad \begin{aligned} L_2 &= a_2(1+A_2T_2) + 2\alpha\gamma_2 a_1 A_1 + \alpha\Pi_2(a_1a_2+A_1A_2), \\ R_2 &= A_2(2+A_2T_2) + 2\alpha\gamma_2 a_1^2 + 2\alpha\Pi_2 a_1 A_2. \end{aligned}$$

It is now possible to solve the simultaneous equation system given by (A.10) and (A.13) for b_{11}, b_{12}, b_{21}, and b_{22}. Thus

$$\text{(A.15)} \qquad \begin{aligned} b_{11} &= \frac{(A_1T_1+\alpha\Pi_1 a_2)R_2 - L_1(2\alpha\gamma_2 a_1+\alpha\Pi_2 A_2)}{R_1R_2 - L_1L_2}, \\ b_{12} &= \frac{(2\alpha\gamma_1 a_2+\alpha\Pi_1 A_1)R_2 - L_1(A_2T_2+\alpha\Pi_2 a_1)}{R_1R_2 - L_1L_2}, \\ b_{21} &= \frac{(2\alpha\gamma_2 a_1+\alpha\Pi_2 A_2)R_1 - L_2(A_1T_1+\alpha\Pi_1 a_2)}{R_1R_2 - L_1L_2}, \text{ and} \\ b_{22} &= \frac{(A_2T_2+\alpha\Pi_2 a_1)R_1 - L_2(2\alpha\gamma_1 a_2+\alpha\Pi_1 A_1)}{R_1R_2 - L_1L_2}. \end{aligned}$$

Substituting (A.15) into (A.9) and (A.1), the two dealers' optimal pricing strategies $p_1^*(x_1,x_2)$ and $p_2^*(x_1,x_2)$ are given by

$$\text{(A.16)} \qquad \begin{aligned} p_1^*(x_1,x_2) = {} & \frac{(A_1T_1+\alpha\Pi_1 a_2)R_2-L_1(2\alpha\gamma_2 a_1+\alpha\Pi_2 A_2)}{R_1R_2-L_1L_2}\, x_1 \\ & + \frac{(2\alpha\gamma_1 a_2+\alpha\Pi_1 A_1)R_2-L_1(A_2T_2+\alpha\Pi_1 a_1)}{R_1R_2-L_1L_2}\, x_2 + b_{13}, \end{aligned}$$

and

$$\text{(A.17)} \qquad \begin{aligned} p_2^*(x_1,x_2) = {} & \frac{(2\alpha\gamma_2 a_1+\alpha\Pi_2 A_2)R_1-L_2(A_1T_1+\alpha\Pi_1 a_2)}{R_1R_2-L_1L_2}\, x_1 \\ & + \frac{(A_2T_2+\alpha\Pi_2 a_1)R_1-L_2(2\alpha\gamma_1 a_2+\alpha\Pi_1 A_1)}{R_1R_2-L_1L_2}\, x_2 + b_{23}. \end{aligned}$$

Furthermore, using (A.10) and (A.13), and defining

$$(A.18)\qquad \begin{aligned} K_1 &= (1+A_1T_1+\alpha\Pi_1a_2)d_1+(2\alpha\gamma_1a_2+\alpha\Pi_1A_1)d_2+A_1h_{b1}-\alpha\Theta_1A_1-\alpha\eta_1a_2, \\ K_2 &= (1+A_2T_2+\alpha\Pi_2a_1)d_2+(2\alpha\gamma_2a_1+\alpha\Pi_2A_2)d_1+A_2h_{b2}-\alpha\Theta_2A_2-\alpha\eta_2a_1, \end{aligned}$$

yields

$$(A.19)\qquad b_{13} = \frac{L_1K_2-K_1R_2}{R_1R_2-L_1L_2}\ ,\quad b_{23} = \frac{L_2K_1-K_2R_1}{R_1R_2-L_1L_2}\ .$$

If (A.16) is substituted into (A.5), after rearranging terms, then

$$(A.20)\qquad \begin{aligned} V^1(x_1,x_2) = {} & [-\tfrac{1}{2}R_1b_{12}^2 + \tfrac{1}{2}T_1a_1^2b_{22}^2 + \alpha\gamma_1(1-A_2b_{22})^2 - \alpha\Pi_1a_1b_{22}(1-A_2b_{22})]\ x_2^2 \\ & - [R_1b_{11}b_{12} + T_1(1-a_1b_{21})a_1b_{22} + 2\alpha\gamma_1A_2b_{21}(1-A_2b_{22}) \\ & - \alpha\Pi_1((1-a_1b_{21})(1-A_2b_{22}) - a_1b_{22}A_2b_{21}]x_1x_2 + [-\tfrac{1}{2}R_1b_{11}^2 \\ & + \tfrac{1}{2}T_1(1-a_1b_{21})^2 + \alpha\gamma_1A_2^2b_{21}^2 - \alpha\Pi_1(1-a_1b_{21})A_2b_{21}]x_1^2 \\ & + \text{const. } x_2 + \text{const. } x_1 + \text{const.} \end{aligned}$$

Obviously the above form is identical with the form of the value function hypothesized earlier, which verifies that the value function has the form given by (A.3).

Equating the corresponding parameters, I obtain

$$(A.21)\qquad \begin{aligned} \gamma_1 &= -\tfrac{1}{2}R_1b_{12}^2 + \tfrac{1}{2}(\alpha\delta_1-h_{c1})a_1^2b_{22}^2 + \alpha\gamma_1(1-A_2b_{22})^2 - \alpha\Pi_1a_1b_{22}(1-A_2b_{22}), \\ \Pi_1 &= -R_1b_{11}b_{12} - (\alpha\delta_1-h_{c1})(1-a_1b_{21})a_1b_{22} - 2\alpha\gamma_1A_2b_{21}(1-A_2b_{22}) \\ & \quad + \alpha\Pi_1((1-a_1b_{21})(1-A_2b_{22})+a_1A_2b_{21}b_{22}), \\ \delta_1 &= -R_1b_{11}^2 + (\alpha\delta_1-h_{c1})(1-a_1b_{21})^2 + 2\alpha\gamma_1A_2^2b_{21}^2 - 2\alpha\Pi_1(1-a_1b_{21})A_2b_{21}. \end{aligned}$$

Analogously it can be shown for the second dealer that

$$(A.22)\qquad \begin{aligned} \gamma_2 &= -\tfrac{1}{2}R_1b_{21}^2 + \tfrac{1}{2}(\alpha\delta_2-h_{c2})a_2^2b_{11}^2 + \alpha\gamma_2(1-A_1b_{11})^2 - \alpha\Pi_2a_2b_{11}(1-A_1b_{11}), \\ \Pi_2 &= -R_2b_{22}b_{21} - (\alpha\delta_2-h_{c2})(1-a_2b_{12})a_2b_{11} - 2\alpha\gamma_2A_1b_{12}(1-A_1b_{11}) \\ & \quad + \alpha\Pi_2((1-a_2b_{12})(1-A_1b_{11})+a_2A_1b_{12}b_{11}), \\ \delta_2 &= -R_2b_{22}^2 + (\alpha\delta_2-h_{c2})(1-a_2b_{12})^2 + 2\alpha\gamma_2A_1^2b_{12}^2 - 2\alpha\Pi_2(1-a_2b_{12})A_1b_{12}. \end{aligned}$$

Since b_{11}, b_{12}, b_{21}, and b_{22} depend on γ_1, Π_1, δ_1, γ_2, Π_2, and δ_2, (A.21) and (A.22) combined form a simultaneous equation system with six equations and six unknowns. Thus all six coefficients can be determined. Due to the complexity of the system, explicit solutions for γ_1, Π_1, δ_1, γ_2, Π_2, and δ_2 will not be derived. But analogous coefficients will be derived for the models with one and two periods in the horizon in the following.

The Model With One Period in the Horizon

In order to interpret the optimal pricing strategies and value functions derived above, I consider the pricing strategies and value functions for the model with one period in the horizon. The results for the infinite-horizon case will be used for this purpose.

Under the assumption that the first dealer faces only one period in the horizon, his decision-making process can be represented by the following functional equation:

$$\text{(A.23)} \quad V_1^1(x_1,x_2)=\max_{p_1}\{p_1(d_1+a_1p_2^*+A_1p_1)-\int H_1(x_1-d_1-a_1p_2^*-A_1p_1-\zeta_1)d\Phi_1(\zeta_1)\}.$$

If the above expression is compared to the functional equation for the infinite-horizon case given by (A.2), it can be seen that the first two terms in (A.2) are identical to the two terms in (A.23). Since the last term in (A.2) represents the first dealer's future decision-making, it has to be equal to zero in the model with only one period in the horizon. Thus (A.21) can be rewritten as (A.4), where $\gamma_1 = \Pi_1 = \eta_1 = \Theta_1 = 0$ and $T_1 = - h_{c1}$.

Using the above information, (A.11) and (A.14) result in

$$
\text{(A.24)} \quad
\begin{aligned}
L_{11} &= a_1(1-A_1h_{c1}) \;, & L_{21} &= a_2(1-A_2h_{c2}) \;, \\
R_{11} &= A_1(2-A_1h_{c1}) \;, & R_{21} &= A_2(2-A_2h_{c2}) \;, \\
K_{11} &= (1-A_1h_{c1})d_1+A_1h_{b1} \;, & K_{21} &= (1-A_2h_{c2})d_2+A_2h_{b2} \;,
\end{aligned}
$$

where the second subscript represents the one-period horizon.

Then from (A.16) and (A.22) the first dealer's optimal pricing strategy $p^*_{11}(x_1,x_2)$ can be derived as

$$
\text{(A.25)} \quad p^*_{11}(x_1,x_2) = -\frac{A_1A_2h_{c1}(2-A_2h_{c2})}{R_{11}R_{21}-L_{11}L_{21}}\,x_1 + \frac{a_1A_2h_{c2}(1-A_1h_{c1})}{R_{11}R_{21}-L_{11}L_{21}}\,x_2 + \frac{c_1}{R_{11}R_{21}-L_{11}L_{21}},
$$

and the second dealer's optimal pricing strategy is given by

$$
\text{(A.26)} \quad p^*_{21}(x_1,x_2) = \frac{a_2A_1h_{c1}(1-A_2h_{c2})}{R_{11}R_{21}-L_{11}L_{21}}\,x_1 - \frac{A_1A_2h_{c2}(2-A_1h_{c1})}{R_{11}R_{21}-L_{11}L_{21}}\,x_2 + \frac{c_2}{R_{11}R_{21}-L_{11}L_{21}} \;,
$$

where

$$
\text{(A.27)} \quad
\begin{aligned}
R_1R_2-L_1L_2 &= A_1A_2(2-A_1h_{c1})(2-A_2h_{c2}) - a_1a_2(1-A_1h_{c1})(1-A_2h_{c2}), \\
c_1 &= a_1(1-A_1h_{c1})((1-A_2h_{c2})d_2+A_2h_{b2}) \\
&\quad - A_2(2-A_2h_{c2})((1-A_1h_{c1})d_1+A_1h_{b1}), \\
c_2 &= a_2(1-A_2h_{c2})((1-A_1h_{c1})d_1+A_1h_{b1}) \\
&\quad - A_1(2-A_1h_{c1})((1-A_2h_{c2})d_2+A_2h_{b2}).
\end{aligned}
$$

Since A_i is negative and a_i, h_{ai}, and $R_{11}R_{21}-L_{11}L_{21}$ are positive, the coefficients of x_1 and x_2 in both dealers' optimal pricing strategies for the one-period horizon are negative. Furthermore, a comparison of

the coefficients of x_1 in both dealers' pricing strategies reveals that the influence of x_1 on $p^*_{11}(x_1,x_2)$ is greater than that on $p^*_{21}(x_1,x_2)$. Analogously, x_2 has a greater impact on $p^*_{21}(x_1,x_2)$ than on $p^*_{11}(x_1,x_2)$. On the other hand, if the coefficients of x_1 and x_2 in the first dealer's pricing strategy $p^*_{11}(x_1,x_2)$ are compared, it can be concluded that the coefficient of x_1 is greater than that of x_2 if $|2A_i h_{ci}| > a_i h_{cj}$. This condition also guarantees that in $p^*_{21}(x_1,x_2)$ the coefficient of x_2 is greater than that of x_1.

Using (A.7) and the knowledge that δ_1, γ_1, and Π_1 are equal to zero, the second derivative of the maximand $G^1_1(x_1,x_2,p_1)$ in (A.23) with respect to p_1 is derived as

$$\text{(A.28)} \qquad Dp_1p_1G^1_1 = A_1(2-A_1h_{c1}) < 0.$$

Similarly, the second dealer's functional equation which can be represented by (A.23), with an appropriate interchange of subscripts, has a maximand $G^2_1(x_1,x_2,p_2)$ whose second derivative with respect to p_2 is

$$\text{(A.29)} \qquad Dp_2p_2G^2_1 = A_2(2-A_2h_{c2}) < 0.$$

Thus the second order conditions are satisfied which guarantee that both dealers maximize their expected profits when following their optimal strategies shown in (A.25) and (A.26).

Equations (A.25) and (A.26) imply

$$\text{(A.30)} \qquad \begin{aligned} b_{11} &= -\frac{A_1A_2h_{c1}(2-A_2h_{c2})}{R_{11}R_{21}-L_{11}L_{21}}, & b_{12} &= \frac{a_1A_2h_{c2}(1-A_1h_{c1})}{R_{11}R_{21}-L_{11}L_{21}}, \\ b_{21} &= \frac{a_2A_1h_{c1}(1-A_2h_{c2})}{R_{11}R_{21}-L_{11}L_{21}}, & b_{22} &= -\frac{A_1A_2h_{c2}(2-A_1h_{c1})}{R_{11}R_{21}-L_{11}L_{21}}. \end{aligned}$$

Substitution of (A.24) and (A.30) into (A.20) yields

$$\text{(A.31)}\quad \begin{aligned} V_1^1(x_1,x_2) = &- \frac{a_1^2A_1A_2^2h_{c2}^2(2-A_1h_{c1})}{2(R_{11}R_{21}-L_{11}L_{21})^2}\,x_2^2 - \frac{a_1A_1A_2h_{c1}h_{c2}(2-A_1h_{c1})M_2}{(R_{11}R_{21}-L_{11}L_{21})^2}\,x_1x_2 \\ &- \frac{h_{c1}(2A_1A_2(2-A_1h_{c1})(2-A_2h_{c2})M_2+a_1^2a_2^2(1-A_2h_{c2})^2)}{2(R_{11}R_{21}-L_{11}L_{21})^2}\,x_1^2 \\ &+ \text{const. } x_2 + \text{const. } x_1 + \text{const.}, \end{aligned}$$

where

$$\text{(A.32)}\quad M_2 = A_1A_2(2-A_2h_{c2}) - a_1a_2(1-A_2h_{c2}) > 0.$$

Thus the first dealer's one period horizon value function $V_1^1(x_1,x_2)$ is the form shown in (A.3) with

$$\text{(A.33)}\quad \begin{aligned} \gamma_1 &= -\frac{a_1^2A_1A_2^2h_{c2}^2(2-A_1h_{c1})}{2(R_{11}R_{21}-L_{11}L_{21})^2}\,, \quad \Pi_1 = -\frac{a_1A_1A_2h_{c1}h_{c2}(2-A_1h_{c1})M_2}{(R_{11}R_{21}-L_{11}L_{21})^2}\,, \\ \delta_1 &= -\frac{h_{c1}(2A_1A_2(2-A_1h_{c1})(2-A_2h_{c2})M_2+a_1^2a_2^2(1-A_2h_{c2})^2)}{(R_{11}R_{12}-L_{11}L_{21})^2}\,. \end{aligned}$$

Since M_2 is positive, γ_1 is also positive, while Π_1 and δ_1 are negative. Hence $V_1^1(x_1,x_2)$ is concave in x_1 and convex in x_2. Analogously, it follows that the second dealer's value function $V_1^2(x_1,x_2)$ is concave in x_2 and convex in x_1.

The Model With Two Periods in the Horizon

Using the results from the model with one period in the horizon, it is now possible to derive the model with two periods in the horizon. If the first dealer faces a horizon consisting of two periods, his decision-making process can be represented by the following functional equation:

$$V_2^1(x_1,x_2) = \max_{p_1} \{p_1(d_1+a_1p_2^*+A_1p_1) - \int H_1(x_1-d_1-a_1p_2^*$$

$$-A_1p_1-\zeta_1)d\Phi_1(\zeta_1) + \alpha \int V_1^1(x_1-d_1-a_1p_2^*-A_1p_1 \tag{A.34}$$

$$-\zeta_1, x_2-d_2-a_2p_1-A_2p_2^*-\zeta_2)d\Phi_{12}(\zeta_1,\zeta_2)\}.$$

The last term in the above functional equation represents the dealer's expected profits in the second and last period. It depends on both dealers' inventories at the beginning of the second period. Since there is only one period left in the dealer's horizon at this point, the first dealer's second decision-making process is represented by the value function $V_1^1(x_1,x_2)$ as shown in (A.23) and (A.31).

As before, the results for the infinite-horizon case are used to derive the optimal pricing strategies and value functions for the model with two periods in the horizon.

Substitution of (A.33) into (A.11) and (A.14) yields

$$\begin{aligned} L_{i2} = {} & \frac{1}{(R_{i1}R_{j1}-L_{i1}L_{j1})^2} \{a_i(1-A_ih_{ci})[A_iA_j^2(A_i(2-A_jh_{cj})^2 \\ & -\alpha a_ia_jA_jh_{cj}^2) + (2A_iA_j(2-A_jh_{cj}) + (1-A_ih_{ci})M_j)(1-A_ih_{ci})M_j] \\ & + \alpha a_iA_ih_{ci}[A_j(2-A_ih_{ci})M_j((A_iA_j-a_ia_j)h_{cj}-4A_i)-a_i^2a_j^2 \\ & (1-A_jh_{cj})^2] - \alpha a_i^2a_jA_iA_j^3h_{cj}^2\}, \end{aligned}$$

$$\begin{aligned} R_{i2} = {} & \frac{1}{(R_{11}R_{21}-L_{11}L_{21})^2} \{2[A_1^2A_2^2(2-A_jh_{cj}^2)^2-\alpha a_1^2a_2^2A_j^2h_{cj}^2 + (2A_1A_2 \\ & (2-A_jh_{cj})+(1-A_ih_{ci})M_j)(1-A_ih_{ci})M_j]-A_ih_{ci}[(R_{11}R_{21} \\ & -L_{11}L_{21})^2 - \alpha A_j(2-A_ih_{ci})M_j(2(A_1A_2-a_1a_2)h_{cj}-4A_i) + \alpha a_1^2a_2^2 \\ & (1-2A_jh_{cj})]\} \end{aligned} \tag{A.35}$$

for i,j=1,2. The sign of L_{i2} cannot be determined, because all terms except the last one are positive. But R_{i2} is clearly negative. This

implies that the second derivative $Dp_1p_1G_2^1$ with respect to p_1 of the maximand in (A.34) is also negative, since $R_1=Dp_1p_1G^1$, as can be deduced from comparing (A.7) with (A.11). Thus I conclude that the second order condition for profit maximization is satisfied when the dealer follows his optimal pricing strategy. This optimal strategy is obtained by substitution of (A.33) and (A.35) into (A.15). Noting that

$$
\begin{aligned}
A_iT_i+\alpha\Pi_ia_j &= -\frac{A_ih_{ci}}{(R_{11}R_{21}-L_{11}L_{21})^2}\{(R_{11}R_{21}-L_{11}L_{21})^2 \\
&\quad - \alpha A_j(2-A_ih_{ci})M_j[(2A_1A_2-a_1a_2)h_{cj}-4A_i] \\
&\quad + \alpha a_1^2a_2^2(1-A_jh_{cj})^2\} > 0, \qquad \text{(A.36)} \\
2\alpha\gamma_ja_i+\alpha\Pi_jA_j &= -\frac{\alpha a_jA_1A_2(2-A_jh_{cj})h_{ci}}{(R_{11}R_{21}-L_{11}L_{21})^2}\{a_1a_2A_ih_{ci}+A_jh_{cj}M_i\} > 0,
\end{aligned}
$$

it can be concluded that the first term $(A_1T_1+\alpha\Pi_1a_2)R_{22}$ in the numerator of b_{11} as shown in (A.15) is negative. Furthermore, the remaining term $-L_{12}(2\alpha\gamma_2a_1+\alpha\Pi_2A_2)$ is also negative except for its last element $-\alpha^2a_1^2a_2^2A_1^2A_2^4h_{c1}h_{c2}^2(2-A_2h_{c2})[a_1a_2A_1h_{c1}+A_2h_{c2}M_1]/(R_{11}R_{21}-L_{11}L_{21})^4$ which is positive. But this positive expression is dominated by one element in $(A_1T_1+\alpha\Pi_1a_2)R_{22}$, namely $-2a_1^2a_2^2A_1^2A_2^2h_{c1}(1-A_2h_{c2})^3(2-A_2h_{c2})(2-A_1h_{c1})M_2$ / $(L_{11}L_{21}-R_{11}R_{21})^4$. Hence it follows that the numerator of b_{11} is negative. It is also possible to show that $R_{12}R_{22}-L_{12}L_{22}$ is positive, implying that b_{11} is negative, but it is omitted here in order to economize on space.

Similarly, substitution of (A.35) and (A.36) into (A.15) yields b_{12}. The first term in its numerator $(2\alpha\gamma_1a_2+\alpha\Pi_2A_1)R_{22}$ is clearly negative. The second term $L_{12}(A_2T_2+\alpha\Pi_2a_1)$ is also negative except for the positive element $-\alpha a_1^2a_2A_1A_2^4h_{c2}^3\{(R_{11}R_{21}-L_{11}L_{21})^2-\alpha A_1(2-A_2h_{c2})M_1$

$[(2A_1A_2-a_1a_2)h_{c1}-4A_2]+\alpha a_1^2a_2^2(1-A_1h_{c1})^2\}/(R_{11}R_{21}-L_{11}L_{21})^2$. But $(2\alpha\gamma_1a_2+\alpha\Pi_1A_1)R_{22}$ consists among other terms of $\alpha a_1^2a_2A_1A_2^4(2-A_1h_{c1})h_{c2}^3$ $\{(R_{11}R_{21}-L_{11}L_{21})^2-\alpha A_1(2-A_{2hc2})M_1[2(A_1A_2-a_1a_2)h_{c1}-4A_2] + \alpha a_1^2a_2^2(1-2A_2h_{c1})\}$ $/(R_{11}R_{21}-L_{11}L_{21})^2$, and $-\alpha a_1A_1^2A_2^2(2-A_1h_{c1})(1-A_2h_{c2})(2-A_2h_{c2})h_{c1}h_{c2}M_1^2$ $\{(1-A_2h_{c2})M_1+2A_1A_2(2-A_1h_{c2})\}/(R_{11}R_{21}-L_{11}L_{21})^2$ which together are absolutely greater than the positive term in $L_{12}(A_2T_2 + \alpha\Pi_2a_1)$. Hence b_{12} is also negative. Since b_{22} and b_{21} are symmetric to b_{11} and b_{12}, respectively, they have the same negative sign.

APPENDIX B
DERIVATION OF THE OPTIMAL PRICING AND ORDERING STRATEGIES FOR THE MODEL WITH INTERDEALER MARKET

The Infinite-Horizon Model

In the following discussion both dealers' optimal pricing strategies $p_1^*(x_1,x_2)$ and $p_2^*(x_1,x_2)$ and ordering strategies $z_1^*(x_1,x_2)$ and $z_2^*(x_1,x_2)$ are derived for the infinite-horizon model with interdealer trading. The derivation follows closely that presented in Appendix A, but the additional assumption that both dealers also trade with each other introduces a second decision variable for each dealer, z_1 and z_2, respectively. To economize on notation, the symbols in Appendix A will be used for corresponding coefficients, e.g., γ_i is the coefficient of x_j, but, of course, the values of these coefficients differ in the two models.

Focusing on the first dealer, I assume that he takes as given his competitor's optimal pricing strategy $p_2^*(x_1,x_2)$ which is hypothesized to be of the form

$$p_2^*(x_1,x_2) = b_{21}x_1 + b_{22}x_2 + b_{23}, \tag{B.1}$$

and his ordering strategy $z_2^*(x_1,x_2)$ which will be represented by

$$z_2^*(x_1,x_2) = s_{21}x_1 + s_{22}x_2 + s_{23}. \tag{B.2}$$

Furthermore, the first dealer is assumed to know both dealers' inventories x_1 and x_2, the joint distribution $\Phi_{12}(\zeta_1,\zeta_2)$, and the marginal distributions $\Phi_1(\zeta_1)$ and $\Phi_2(\zeta_2)$ of the random variables ζ_1 and ζ_2.

The first dealer's decision-making process can then be described by the functional equation

$$V^1(x_1,x_2) = \max_{p_1,z_1}\{p_1(d_1+a_1p_2^*+A_1p_1+z_2^*) - p_2^*z_1 - \int H_1(x_1+z_1-z_2^*-d_1-a_1p_2^*$$

$$-A_1p_1-\zeta_1)d\Phi_1(\zeta_1) + \alpha\int V^1(x_1+z_1-z_2^*-d_1-a_1p_2^*-A_1p_1-\zeta_1, x_2+z_2^* \tag{B.3}$$

$$-z_1-d_2-a_2p_1-A_2p_2^*-\zeta_2)d\Phi_{12}(\zeta_1,\zeta_2)\}.$$

As in Appendix A it will be hypothesized that the value function shown above has the form

$$V^1(x_1,x_2) = \tfrac{1}{2}\delta_1x_1^2 + \Pi_1x_1x_2 + \gamma_1x_2^2 + \Theta_1x_1 + \eta_1x_2 + \rho_1. \tag{B.4}$$

Using this hypothesized form for the next period's value function in (B.3), and substituting (2.2) into (B.3), I obtain

$$V^1(x_1,x_2) = \max_{p_1,z_1}\{p_1(d_1+a_1p_2^*+A_1p_1+z_2^*) - p_2^*z_1 - h_{a1} - h_{b1}(x_1+z_1-z_2^*-d_1-a_1p_2^*$$

$$-A_1p_1) + \tfrac{1}{2}T_1(x_1+z_1-z_2^*-d_1-a_1p_2^*-A_1p_1)^2 + \tfrac{1}{2}T_1\sigma_{\zeta1}^2 + \alpha\Pi_1(x_1+z_1-z_2^*$$

$$-d_1-a_1p_2^*-A_1p_1)(x_2+z_2^*-z_1-d_2-a_2p_1-A_2p_2^*) + \alpha\Pi_1\sigma_{\zeta1\zeta2}^2 \tag{B.5}$$

$$+\alpha\gamma_1(x_2+z_2-z_1-d_2-a_2p_1-A_2p_2^*)^2 + \alpha\gamma_1\sigma_{\zeta2}^2 + \alpha\Theta_1(x_1+z_1-z_2^*-d_1$$

$$-a_1p_2^*-A_1p_1) + \alpha\eta_1(x_2+z_2^*-z_1-d_2-a_2p_1-A_2p_2^*) + \alpha\rho_1\},$$

where $T_1 = \alpha\delta_1-h_{c1}$. Defining

$$k_{21} = a_1b_{21}+s_{21}, \quad k_{22} = a_1b_{22}+s_{22}, \quad k_{23} = a_1b_{23}+s_{23}, \text{ and}$$

$$r_{21} = s_{21}-A_2b_{21}, \quad r_{22} = s_{22}-A_2b_{22}, \quad r_{23} = s_{23}-A_2b_{23}, \tag{B.6}$$

I then obtain by substituting (B.1), (B.2), and (2.2) into (B.5), and after rearranging terms,

$$
\begin{aligned}
V^1(x_1,x_2) = \max_{p_1,z_1} \{ & [\tfrac{1}{2}A_1(2+A_1T_1)+\alpha\Pi_1a_2A_1+\alpha\gamma_1a_2^2]p_1^2 + [-A_1T_1+\alpha\Pi_1(A_1-a_2) \\
& + 2\alpha\gamma_1a_2]p_1z_1 + [\tfrac{1}{2}T_1-\alpha\Pi_1+\alpha\gamma_1]z_1^2 + [(k_{21}-A_1T_1(1-k_{21}) \\
& - \alpha\Pi_1(a_1(1-k_{21})+A_1r_{21})-2\alpha\gamma_1a_2r_{21})x_1 + (k_{22}+A_1T_1k_{22}+\alpha\Pi_1(a_2k_{22} \\
& - A_1(1+r_{22}))-2\alpha\gamma_1a_2(1+r_{22}))x_2 + (1+A_1T_1+\alpha\Pi_1a_2)(k_{23}+d_1) \\
& + (\alpha\Pi_1A_1+2\alpha\gamma_1a_2)(d_2-r_{23}) + A_1h_{b1}-\alpha\theta_1A_1-\alpha\eta_1a_2]p_1 + [(-b_{21} \\
& + T_1(1-k_{21})+\alpha\Pi_1(r_{21}+k_{21}-1)-2\alpha\gamma_1r_{21})x_1 + (-b_{22}-T_1k_{22}+\alpha\Pi_1(1 \\
& + r_{22}+k_{22})-2\alpha\gamma_1(1+r_{22}))x_2 - b_{23}-(T_1-\alpha\Pi_1)(k_{23}+d_1) + (\alpha\Pi_1 \\
& - 2\alpha\gamma_1)(r_{23}-d_2)-h_{b1}+\alpha\theta_1-\alpha\eta_1]z_1 + [\tfrac{1}{2}T_1(1-k_{21})^2+\alpha\Pi_1(1-k_{21})r_{21} \\
& + \alpha\gamma_1r_2^2]x_1^2 + [-T_1(1-k_{21})k_{22}+\alpha\Pi_1((1-k_{21})(1+r_{22})-k_{22}r_{21}) \\
& + 2\alpha\gamma_1(1+r_{22})r_{21}]x_1x_2 + [\tfrac{1}{2}T_1k_{22}^2-\alpha\Pi_1k_{22}(1+r_{22})+\alpha\gamma_1(1+r_{22})^2]x_2^2 \\
& + \text{const. } x_1 + \text{const. } x_2 + \text{const.}\}.
\end{aligned} \tag{B.7}
$$

Defining $G^1(x_1,x_2,p_1,z_1)$ as the maximand in the functional equation above, the derivative of this maximand with respect to p_1 is given by

$$
\begin{aligned}
Dp_1G^1 = {} & [A_1(2+A_1T_1)+2\alpha\Pi_1a_2A_1+2\alpha\gamma_1a_2^2]p_1 + [-A_1T_1+\alpha\Pi_1(A_1-a_2) \\
& + 2\alpha\gamma_1a_2]z_1 + [k_{21}-A_1T_1(1-k_{21})-\alpha\Pi_1(a_2(1-k_{21})+A_1r_{21}) \\
& - 2\alpha\gamma_1a_2r_{21}]x_1 + [(1-A_1T_1)k_{22} + \alpha\Pi_1(a_2k_{22}-A_1(1+r_{22})) \\
& - 2\alpha\gamma_1a_2(1+r_{22})]x_2 + (1+A_1T_1+\alpha\Pi_1a_2)(k_{23}+d_1) + (\alpha\Pi_1A_1 \\
& + 2\alpha\gamma_1a_2)(d_2-r_{23}) + A_1h_{b1} - \alpha\theta_1A_1 - \alpha\eta_1a_2,
\end{aligned} \tag{B.8}
$$

and the derivative of the same maximand with respect to z_1 is

$$
\begin{aligned}
Dz_1G^1 = {} & [T_1-2\alpha\Pi_1+2\alpha\gamma_1]z_1 + [-A_1T_1+\alpha\Pi_1(A_1-a_2)+2\alpha\gamma_1a_2]p_1 + [-b_{21} \\
& + T_1(1-k_{21})+\alpha\Pi_1(r_{21}+k_{21}-1)-2\alpha\gamma_1r_{21}]x_1 + [-b_{22}-T_1k_{22} \\
& + \alpha\Pi_1(1+r_{22}+k_{22})-2\alpha\gamma_1(1+r_{22})]x_2 - b_{23} - (T_1-\alpha\Pi_1)(k_{23}+d_1) \\
& + (\alpha\Pi_1-2\alpha\gamma_1)(r_{23}-d_2) - h_{b1} + \alpha\theta_1 - \alpha\eta_1.
\end{aligned} \tag{B.9}
$$

Taking the derivative of Dp_1G^1 and Dz_1G^1 with respect to p_1 and z_1 yields

(B.10)
$$Dp_1p_1G^1 = A_1(2+A_1T_1) + 2\alpha\Pi_1a_2A_1 + 2\alpha\gamma_1a_2^2,$$
$$Dz_1z_1G^1 = T_1 - 2\alpha\Pi_1 + 2\alpha\gamma_1, \text{ and}$$
$$Dp_1z_1G^1 = Dz_1p_1G^1 = -A_1T_1 + \alpha\Pi_1(A_1-a_2) + 2\alpha\gamma_1a_2.$$

Thus, in order to guarantee that the first dealer maximizes the expected present value of his infinite-horizon profits when using his optimal strategies $p_1^*(x_1,x_2)$ and $z_1^*(x_1,x_2)$, the following conditions must be satisfied:

(B.11)
$$Dp_1p_1G^1 = A_1(2+A_1T_1) + 2\alpha\Pi_1a_2A_1 + 2\alpha\gamma_1a_2^2 < 0,$$
$$Dz_1z_1G^1 = T_1 - 2\alpha\Pi_1 + 2\alpha\gamma_1 < 0, \text{ and}$$
$$D_1 = 2A_1T_1 + 4\alpha(\gamma_1-\Pi_1)A_1 + (2\alpha\gamma_1T_1-\alpha^2\Pi_1^2)(A_1+a_2)^2 > 0,$$

noting that

(B.12)
$$D_1 = Dp_1p_1G^1 \cdot Dz_1z_1G^1 - (Dp_1z_1G^1)^2.$$

Setting both Dp_1G^1 and Dz_1G^1 equal to zero and solving simultaneously for p_1 and z_1, the first dealer's optimal pricing strategy $p_1^*(x_1,x_2)$ is derived as

(B.13)
$$p_1^*(x_1,x_2) = b_{11}x_1 + b_{12}x_2 + b_{13},$$

and his optimal ordering strategy as

(B.14)
$$z_1^*(x_1,x_2) = s_{11}x_1 + s_{12}x_2 + s_{13},$$

where

$$b_{11} = \frac{1}{D_1}[Q_{1b}b_{21}+Q_{1s}s_{21}+Q_{11}], \quad b_{12} = \frac{1}{D_1}[Q_{1b}b_{22}+Q_{1s}s_{22}+Q_{11}],$$

(B.15)

$$s_{11} = \frac{1}{D_1}[Z_{1b}b_{21}+Z_{1s}s_{21}+Q_{12}], \quad s_{12} = \frac{1}{D_1}[Z_{1b}b_{22}+Z_{1s}s_{22}+Q_{13}],$$

and

$$Q_{ib} = (A_i-a_i)T_i - (2\alpha\gamma_iT_i-\alpha^2\Pi_i^2)(A_i+a_j)(A_j+a_i)$$
$$- 2\alpha\gamma_i(a_i+a_j) - \alpha\Pi_i(A_i-2a_i-a_j),$$
$$Q_{is} = -(T_i - 2\alpha\Pi_i + 2\alpha\gamma_i) = -Dz_iz_iG^i,$$
$$Z_{ib} = A_i(2+(A_i+a_i)T_i) - (2\alpha\gamma_iT_i-\alpha^2\Pi_i^2)(A_i+a_j)(A_iA_j-a_ia_j)$$
$$- 2\alpha\gamma_i(2A_iA_j-a_ia_j-a_j^2) + \alpha\Pi_i(2A_i(A_j+a_j)-a_i(A_i+a_j)),$$

(B.16)
$$Z_{is} = A_iT_i + (2\alpha\gamma_iT_i-\alpha^2\Pi_i^2)(A_i+a_j)^2$$
$$+ 2\alpha\gamma_i(2A_i+a_j) - \alpha\Pi_i(3A_i+a_j),$$
$$Q_{i1} = (2\alpha\gamma_iT_i - \alpha^2\Pi_i^2)(A_i+a_j),$$
$$Q_{i2} = -2A_iT_i - (2\alpha\gamma_iT_i-\alpha^2\Pi_i^2)a_j(A_i+a_j) + 2\alpha\Pi_iA_i, \text{ and}$$
$$Q_{i3} = (2\alpha\gamma_iT_i-\alpha^2\Pi_i^2)A_i(A_i+a_j) + 4\alpha\gamma_iA_i - 2\alpha\Pi_iA_i$$

for $i=1$ and $j=2$.

In analogy to the derivation of the first dealer's optimal pricing and ordering strategies given by (B.1) through (B.16), I can derive the second dealer's optimal pricing strategy $p_2^*(x_1,x_2)$ and ordering strategy $z_2^*(x_1,x_2)$ as shown in (B.1) and (B.2), where

$$b_{21} = \frac{1}{D_2}[Q_{2b}b_{11}+Q_{2s}s_{11}+Q_{21}], \quad b_{22} = \frac{1}{D_2}[Q_{2b}b_{12}+Q_{2s}s_{12}+Q_{21}],$$

(B.17)

$$s_{21} = \frac{1}{D_2}[Z_{2b}b_{11}+Z_{2s}s_{11}+Q_{23}], \quad s_{22} = \frac{1}{D_2}[Z_{2b}b_{12}+Z_{2s}s_{12}+Q_{22}]$$

with

$$\text{(B.18)}\quad \begin{aligned} D_2 &= Dp_2p_2G^2 \cdot Dz_2z_2G^2 - (Dp_2z_2G^2)^2 \\ &= 2A_2T_2 + (2\alpha\gamma_2T_2-\alpha^2\Pi_2^2)(A_2+a_1)^2 + 4\alpha(\gamma_2-\Pi_2)A_2 \end{aligned}$$

and (B.16) for i=2 and j=1.

The four equations on the left-hand side of (B.15) and (B.17) form a simultaneous equation system that uniquely determines b_{11}, s_{11}, b_{21}, and s_{21}. Similarly, the four equations on the right-hand side of (B.15) and (B.17) jointly determine b_{12}, s_{12}, b_{22}, and s_{22}. Defining

$$\text{(B.19)}\quad \begin{aligned} R_{11} &= Q_{1b}Q_{2s}+Q_{1s}Z_{2s}\ , & R_{12} &= Z_{1b}Q_{2b}+Z_{1s}Z_{2b}\ , \\ J_{11} &= D_1D_2 - (Q_{1b}Q_{2b}+Q_{1s}Z_{2b}), & J_{12} &= D_1D_2 - (Z_{1b}Q_{2s}+Z_{1s}Z_{2s})\ , \\ N_{11} &= Z_{1b}Q_{21}+Z_{1s}Q_{23}+Q_{12}D_2\ , & N_{12} &= Q_{1b}Q_{21}+Q_{1s}Q_{23}+Q_{11}D_2, \end{aligned}$$

the following expressions are derived from the first simultaneous equation system

$$b_{11} = \frac{R_{11}N_{11}+N_{12}J_{12}}{J_{11}J_{12}-R_{11}R_{12}}\ , \qquad s_{11} = \frac{J_{11}N_{11}+N_{12}R_{12}}{J_{11}J_{12}-R_{11}R_{12}}\ ,$$

$$\text{(B.20)}\quad b_{21} = \frac{Q_{2b}(R_{11}N_{11}+N_{12}J_{12})+Q_{2s}(J_{11}N_{11}+N_{12}R_{12})+Q_{21}(J_{11}J_{12}-R_{11}R_{12})}{D_2(J_{11}J_{12}-R_{11}R_{12})}\ ,$$

$$s_{21} = \frac{Z_{2b}(R_{11}N_{11}+N_{12}J_{12})+Z_{2s}(J_{11}N_{11}+N_{12}R_{12})+Q_{23}(J_{11}J_{12}-R_{11}R_{12})}{D_2(J_{11}J_{12}-R_{11}R_{12})}\ ,$$

and from the second simultaneous equation system I obtain

$$b_{12} = \frac{R_{11}M_{11}+J_{12}M_{12}}{J_{11}J_{12}-R_{11}R_{12}}\ , \qquad s_{12} = \frac{R_{12}M_{12}+J_{11}M_{11}}{J_{11}J_{12}-R_{11}R_{12}}\ ,$$

$$\text{(B.21)}\quad b_{22} = \frac{Q_{2b}(R_{11}M_{11}+J_{12}M_{12})+Q_{2s}(R_{12}M_{12}+J_{11}M_{11})+Q_{21}(J_{11}J_{12}-R_{11}R_{12})}{D_2(J_{11}J_{12}-R_{11}R_{12})}\ ,$$

$$s_{22} = \frac{Z_{2b}(R_{11}M_{11}+J_{12}M_{12})+Z_{2s}(R_{12}M_{12}+J_{11}M_{11})+Q_{22}(J_{11}J_{12}-R_{11}R_{12})}{D_2(J_{11}J_{12}-R_{11}R_{12})},$$

where

$$\begin{aligned} M_{11} &= Z_{1b}Q_{21}+Z_{1s}Q_{22}+Q_{13}D_2, \\ M_{12} &= Q_{1b}Q_{21}+Q_{1s}Q_{22}+Q_{11}D_2. \end{aligned} \tag{B.22}$$

Substituting (B.20) and (B.21) into (B.13) and (B.14), I can write the first dealer's optimal pricing strategy as

$$p_1^*(x_1,x_2) = \frac{R_{11}N_{11}+N_{12}J_{12}}{J_{11}J_{12}-R_{11}R_{12}}x_1 + \frac{R_{11}M_{11}+M_{12}J_{12}}{J_{11}J_{12}-R_{11}R_{12}}x_2 + b_{13} \tag{B.23}$$

and his optimal ordering strategy as

$$z_1^*(x_1,x_2) = \frac{J_{11}N_{11}+N_{12}R_{12}}{J_{11}J_{12}-R_{11}R_{12}}x_1 + \frac{R_{12}M_{12}+M_{11}J_{11}}{J_{11}J_{12}-R_{11}R_{12}}x_2 + s_{13}. \tag{B.24}$$

Furthermore, b_{13} and s_{13} can be expressed explicitly by

$$b_{13} = \frac{C_{b1}O_{s1}+C_{s1}W_{b1}}{O_{b1}O_{s1}(1-W_{b1}W_{s1})}, \qquad s_{13} = \frac{C_{s1}O_{b1}+C_{b1}W_{s1}}{O_{b1}O_{s1}(1-W_{b1}W_{s1})}, \tag{B.25}$$

where, for i=1 and j=2,

$$\begin{aligned} W_{i1} &= -(1+a_iT_i) - \alpha\Pi_i(A_j-a_i) + 2\alpha\gamma_iA_j, \\ W_{i2} &= (\alpha\Pi_i-T_i)d_i - (\alpha\Pi_i-2\alpha\gamma_i)d_j - h_{bi} + \alpha\theta_i - \alpha\eta_i \\ O_{bi} &= D_iD_j - [Dp_iz_iG^iW_{i1}-Dz_iz_iG^iL_i][Dp_jz_jG^jW_{j1}-Dz_jz_jG^jL_j] \\ &\quad + Dz_iz_iG^i[Dp_jz_jG^jL_j-Dp_jp_jG^jW_{j1}], \\ W_{bi} &= -[Dp_iz_iG^iW_{i1}-Dz_iz_iG^iL_i]Dz_jz_jG^j-Dz_iz_iG^j[D_j+Dp_jz_jG^2], \\ C_{b1} &= [Dp_iz_iG^iW_{i1}-Dz_iz_iG^iL_i][Dp_jz_jG^jW_{j2}-Dz_jz_jG^jK_j] \\ &\quad - Dz_iz_iG^i[Dp_jz_jG^jK_j-Dp_jp_jG^jW_{j2}] + D_j[Dp_iz_iG^iW_{i2}-Dz_iz_iG^iK_i], \\ O_{si} &= D_iD_j + [Dp_iz_iG^iL_i-Dp_ip_iG^iW_{i1}]Dz_jz_jG^j \\ &\quad - [D_i+Dp_iz_iG^i][D_j+Dp_jz_jG^j], \end{aligned} \tag{B.26}$$

$$W_{si} = [Dp_iz_iG^iL_i - Dp_ip_iG^iW_{i1}][Dp_jz_jG^jW_{j1} - Dz_jz_jG^jL_j]$$
$$+ [D_i + Dp_iz_iG^i][Dp_jz_jG^jL_j - Dp_jp_jG^jW_{j1}],$$
$$C_{si} = [Dp_iz_iG^iL_i - Dp_ip_iG^iW_{i1}][Dp_jz_jG^jW_{j2} - Dz_jz_jG^jK_j]$$
$$+ [D_i + Dp_iz_iG^i][Dp_jz_jG^jK_j - Dp_jp_jG^jW_{j2}]$$
$$+ D_j[Dp_iz_iG^iK_i - Dp_ip_iG^iW_{i2}],$$

and L_i and K_i are defined as in (A.11) and (A.18), respectively. If (B.1), (B.2), (B.13), and (B.14) are substituted into (B.5), a functional equation of the form shown in (B.4) is obtained which confirms the hypothesis made earlier about the particular form of the value function. Furthermore, equating corresponding coefficients yields

$$\begin{aligned}
\delta_1 = {} & 2[A_1b_{11}^2 + b_{11}(a_1b_{21} + s_{21}) - b_{21}s_{11} + \tfrac{1}{2}(\alpha\delta_1 - h_{c1})(1 + s_{11} - s_{21} \\
& -A_1b_{11} - a_1b_{21})^2 + \alpha\Pi_1(1 + s_{11} - s_{21} - A_1b_{11} - a_1b_{21})(-s_{11} + s_{21} \\
& -a_2b_{11} - A_2b_{21}) + \alpha\gamma_1(-s_{11} + s_{21} - a_2b_{11} - A_2b_{21})^2], \\
\Pi_1 = {} & 2A_1b_{11}b_{12} + b_{11}(a_1b_{22} + s_{22}) + b_{12}(a_1b_{21} + s_{21}) - b_{21}s_{12} \\
& - b_{22}s_{11} + (\alpha\delta_1 - h_{c1})(1 + s_{11} - s_{21} - A_1b_{11} - a_1b_{21})(s_{12} - s_{22} - A_1b_{12} \\
& -a_1b_{22}) + \alpha\Pi_1[(1 + s_{11} - s_{21} - A_1b_{11} - a_1b_{21})(1 - s_{12} + s_{22} - a_2b_{12} \\
& -A_2b_{22}) + (s_{12} - s_{22} - A_1b_{12} - a_1b_{22})(-s_{11} + s_{21} - a_2b_{11} - A_2b_{21})] \\
& + 2\alpha\gamma_1(-s_{11} + s_{21} - a_2b_{11} - A_2b_{21})(1 - s_{12} + s_{22} - a_2b_{12} - A_2b_{22}), \\
\gamma_1 = {} & A_1b_{12}^2 + b_{12}(a_1b_{22} + s_{22}) - b_{22}s_{12} + \tfrac{1}{2}(\alpha\delta_1 - h_{c1})(s_{12} - s_{22} \\
& -A_1b_{12} - a_1b_{22})^2 + \alpha\Pi_1(s_{12} - s_{22} - A_1b_{12} - a_1b_{22})(1 - s_{12} - s_{22} - a_2b_{12} \\
& -A_2b_{22}) + \alpha\gamma_1(1 - s_{12} + s_{22} - a_2b_{12} - A_2b_{22})^2.
\end{aligned} \tag{B.27}$$

Changing subscripts accordingly, an equation system analogous to (B.27) can be derived for the coefficients δ_2, Π_2, and γ_2 of the second dealer's value function $V^2(x_1,x_2)$. Since b_{ij} and s_{ij} (i,j=1,2) as shown in (B.20) and (B.21) are all functions of δ_1, Π_1, γ_1 and δ_2, Π_2, γ_2,

(B.27) and the analogous system for the second dealer together form a simultaneous equation system with six equations and the six unknowns δ_1, Π_1, γ_1, δ_2, Π_2, γ_2. Thus these six coefficients can be determined by solving a system of six equations. As in the case of the Model Without Interdealer Market, explicit solutions for the six coefficients will not be derived due to the complexity of the system. But, to assist in interpretation, analogous coefficients will be derived for the model with one period in the horizon.

The Model With One Period in the Horizon

Analogously to Appendix A, I examine the pricing and ordering strategies as well as the value functions for the model with one period in the horizon. This assists in interpreting the results for the infinite-horizon case which have been derived in the previous section and will be used in the present discussion.

Assuming that the first dealer faces only one period in the horizon, his decision-making process can be represented by the following functional equation

$$V_1^1(x_1,x_2) = \max_{p_1,z_1}\{p_1(d_1+a_1p_2^*+A_1p_1+z_2^*) - p_2^*z_1 - \int H_1(x_1+z_1-z_2^* -d_1-a_1p_2^*-A_1p_1-\zeta_1)d\Phi_1(\zeta_1)\}. \tag{B.28}$$

This expression is identical to (B.3) except for an additional term in the right-hand side of (B.3) which represents the expected present value of the first dealer's future profits. In the case of one period in the horizon, this present value is obviously zero. Alternately I can rewrite (B.28) as (B.5) with $\gamma_1 = \Pi_1 = \eta_1 = \Theta_1 = 0$ and $T_1 = -h_{c1}$.

Thus (B.16) can be modified for this special case to

(B.29) $$Q_{ib} = -(A_i-a_i)h_{ci}\ , \qquad Q_{is} = h_{ci}\ , \qquad Q_{i1} = Q_{i3} = 0\ ,$$
$$Z_{ib} = A_i(2-(A_i+a_i)h_{ci})\ ,\ Z_{is} = -A_ih_{ci}\ ,\ Q_{i2} = 2A_ih_{ci}.$$

From (B.11), (B.18), and (B.19) results

(B.30)
$$D_1 = -2A_1h_{c1}\ ,\ D_2 = -2A_2h_{c2}\ ,\ R_{11} = -(A_1+A_2-a_1)h_{c1}h_{c2}>0,$$
$$R_{12} = A_1[(A_1+a_1)(A_2-a_2)+A_2(A_2+a_2)]h_{c1}h_{c2} - 2A_1(A_2-a_2)h_{c2} - 2A_1A_2h_{c1}<0,$$
$$J_{11} = [3A_1A_2+A_2^2+A_1a_2+A_2a_1+A_2a_2-a_1a_2-\frac{2A_2}{h_{c2}}]h_{c1}h_{c2}>0,$$
$$J_{12} = A_1[3A_2+A_1+a_1-\frac{2}{h_{c1}}]h_{c1}h_{c2}>0,$$
$$N_{11} = -4A_1A_2h_{c1}h_{c2}<0\ , \quad N_{12} = 0.$$

Substitution of (B.30) into (B.20) yields

$$b_{11} = \frac{4A_1A_2(A_1+A_2-a_1)h_{c1}^2h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}}\ ,\ b_{21} = \frac{4A_1A_2(2A_1+A_2-\frac{1}{h_{c2}})h_{c1}^2h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}}\ ,$$

(B.31)
$$s_{11} = -\frac{4A_1A_2[3A_1A_2+A_2^2+A_1a_2+A_2a_1+A_2a_2-a_1a_2-\frac{2A_2}{h_{c2}}]h_{c1}^2h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}}\ ,$$
$$s_{21} = -\frac{4A_1A_2[A_2(A_1+a_1)h_{c2}+A_1-a_1]h_{c1}^2h_{c2}}{J_{11}J_{12}-R_{11}R_{12}}\ .$$

Similarly, using (B.18) and (B.22), I can write

(B.32) $$M_{11} = -\ 2A_1A_2h_{c1}h_{c2}\ , \qquad M_{12} = 2A_2h_{c1}h_{c2}\ ,$$

and, substituting (B.30) and (B.32) into (B.21), I obtain

$$b_{12} = \frac{4A_1A_2(2A_2+A_1-\frac{1}{h_{c1}})h_{c1}^2h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}}\,, \quad b_{22} = \frac{4A_1A_2(A_2+A_1-a_2)h_{c1}^2h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}}\,,$$

$$\text{(B.33)} \qquad s_{12} = -\frac{4A_1A_2[A_1(A_2+a_2)h_{c1}+(A_2-a_2)]h_{c1}h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}}\,,$$

$$s_{22} = -\frac{4A_1A_2[3A_1A_2+A_1^2+A_2a_1+A_1a_2+A_1a_1-a_1a_2-\frac{2A_1}{h_{c1}}]h_{c1}^2h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}}\,.$$

Since J_{11}, J_{12}, and R_{11} are positive, while R_{12} is negative, the denominator of all coefficients in (B.31) and (B.33) is positive. Hence b_{11}, b_{21}, s_{11}, b_{12}, b_{22} and s_{22} are negative, while the sign of s_{21} and s_{12} cannot be determined. It can also be observed that $|b_{ii}|<|b_{ij}|$ and $|b_{ii}|<|b_{ji}|$ for i,j = 1,2. Furthermore, noting that

$$\text{(B.34)} \qquad \begin{aligned} J_{11}J_{12}-R_{11}R_{12} = 4A_1A_2h_{c1}h_{c2}\{&[3A_1A_2+A_1^2+A_2^2+A_1a_1+A_2a_2+A_1a_2+A_2a_1 \\ &-a_1a_2]h_{c1}h_{c2} - (2A_2+A_1)h_{c1}-(2A_1+A_2)h_{c2}+1\}, \end{aligned}$$

it can be seen that $J_{11}J_{12}-R_{11}R_{12}$ is perfectly symmetric, that means that it will not change its value if we interchange the first and the second dealers' subscripts. Thus it is now possible to verify that b_{22}, b_{21}, s_{22} and s_{21} are symmetric to b_{11}, b_{12}, s_{11}, and s_{12}, respectively. Therefore it suffices to examine one dealer's optimal strategies in order to understand both dealers' optimal behavior.

Substituting (B.31) and (B.33) into (B.23) yields the first dealer's optimal pricing strategy

$$\text{(B.35)} \qquad p_{11}^*(x_1,x_2) = \frac{4A_1A_2h_{c1}^2h_{c2}^2}{J_{11}J_{12}-R_{11}R_{12}}\{(A_1+A_2-a_1)x_1+(2A_2+A_1-\frac{1}{h_{c2}})x_2\} + b_{13}.$$

Similarly, if (B.31) and (B.33) are substituted into (B.24), the first dealer's optimal ordering strategy is derived as

(B.36)
$$z^*_{11}(x_1,x_2) = -\frac{4A_1A_2[3A_1A_2+A_2^2+A_1a_2+A_2a_1+A_2a_2-a_1a_2-\frac{2A_2}{h_{c2}}]h^2_{c1}h^2_{c2}}{J_{11}J_{12}-R_{11}R_{12}}x_1$$
$$-\frac{4A_1A_2[A_1(A_2+a_2)h_{c1}+(A_2-a_2)]h_{c1}h^2_{c2}}{J_{11}J_{12}-R_{11}R_{12}}x_2+s_{13}.$$

Using (B.11) and the knowledge that δ_1, Π_1, and γ_1 equal zero, it is verified that the first dealer maximizes his expected profits by following his optimal pricing and ordering strategies, since

(B.37)
$$Dp_1p_1G^1_1 = A_1(2-A_1h_{c1}) < 0,$$
$$Dz_1z_1G^1_1 = -h_{c1} < 0,$$
$$D_1 = Dp_1p_1G^1_1 \cdot Dz_1z_1G^1_1 - (Dp_1z_1G^1_1)^2 = -2A_1h_{c1} > 0.$$

In addition, using (B.10) and (B.30) I can rewrite (B.26) as

(B.38)
$$W_{i1} = -(1-a_ih_{ci}) < 0, \qquad W_{i2} = h_{ci}d_i-h_{bi} > 0,$$
$$O_{bi} = (3A_1A_2+A_2^2+A_2a_1+A_1a_2+A_2a_2-a_1a_2-\frac{2A_2}{h_{c2}})h_{ci}h_{cj} > 0,$$
$$W_{bi} = -(A_i+A_j-a_i)h_{ci}h_{cj} > 0,$$
$$C_{bi} = -(A_i+A_j-a_i)h_{ci}h_{cj}d_j-2A_jh_{ci}(h_{cj}d_i-h_{bi}) > 0,$$
$$O_{si} = 3A_iA_jh_{ci}h_{cj}-A_i(2-(A_i+a_i)h_{ci}) > 0,$$
$$W_{si} = -A_i(A_j-a_j)(2-(A_i-a_i)h_{ci})h_{cj}-A_iA_j(2-(A_j+a_j)h_{cj})h_{ci} < 0,$$
$$C_{si} = A_i(2-(A_i+a_i)h_{ci})h_{cj}d_j + A_iA_j(h_{ci}h_{cj}(d_j+2d_i)$$
$$-2(h_{ci}h_{bj}+ah_{cj}h_{bi})).$$

Thus the denominator of the constants b_{13} and s_{13} shown in (B.25) is positive. Furthermore, if (B.38) is substituted into (B.25), then

(B.39)
$$b_{13} = - \frac{2A_2h_{c1}}{O_{b1}O_{s1}(1-W_{b1}W_{s1})} [A_1(A_1+A_2-a_1)((2d_2+d_1)h_{c1}h_{b2} - h_{c1}h_{b2}-2h_{c2}h_{b1})h_{c2} + (h_{c2}d_1-h_{b2})O_{s1}] > 0,$$

while the sign of s_{13} cannot be determined without making additional assumptions about the relative sizes of the parameters involved.

In order to examine the first dealer's value function in the one-period-horizon model, I make use of (B.31) and (B.33). Then I derive from (B.27) the respective coefficients for x_1^2, x_1x_2, and x_2^2 as

(B.40)
$$\delta_1 = \frac{32A_1^2(A_1+A_2-a_1)(A_1+a_1-\frac{1}{h_{c2}})h_{c1}^4h_{c2}^4}{(J_{11}J_{12}-R_{11}R_{12})^2} - 2b_{21}s_{11} - h_{c1}(1+s_{11}-s_{21}-A_1b_{11}-a_1b_{21})^2,$$

$$\Pi_1 = \frac{16A_1^2A_2^2h_{c1}^3h_{c2}^3}{(J_{11}J_{12}-R_{11}R_{12})^2}\{[A_1(6A_1A_2+4A_2a_1+2A_1a_2)+A_2^2(A_2+a_1) + A_1^2(A_1+a_1)+4A_1A_2^2-A_1A_2a_2-2A_2a_1a_2-A_1a_2^2-A_2a_2^2]h_{c1}h_{c2} - [5A_1A_2+A_1^2+2A_2^2+A_1a_2-2A_2a_2]h_{c1} - 2A_1^2h_{c2} + 2A_1\},$$

$$\gamma_1 = \frac{8A_1^2A_2^2h_{c1}^2h_{c2}^4}{(J_{11}J_{12}-R_{11}R_{12})^2}\{-2A_1(A_2+a_2)^2h_{c1}^2 + [10A_1A_2+3A_1^2 + A_2^2-2A_1a_2-2A_2a_2+a_2^2]h_{c2}-A_1\}.$$

Recalling that A_i, b_{ji}, $s_{ii} < 0$, a_i, $h_{ci} > 0$, and $|A_i| > a_j$ for i,j = 1,2, I conclude that δ_1 and Π_1 are negative, while γ_1 is positive. Hence the first dealer's one-period-horizon value function $V_1^1(x_1,x_2)$ is

concave in x_1 and convex in x_2. Similarly it follows that the second dealer's value function $V_1^2(x_1,x_2)$ is convex in x_1 and concave in x_2.

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BIOGRAPHICAL SKETCH

Barbara Kauffmann was born in Cologne, Federal Republic of Germany, in 1960. After her graduation from high school, the Schubart-Gymnasium Aalen, in 1979, she lived in Paris, France, and Madrid, Spain, to study French and Spanish.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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